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# ECONOMICS OF A MONETARY UNION

PhD Thesis  
written under the supervision of:  
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dr. Burak R. Uras

Tilburg, June 2018



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# Chapter 1

## Introduction

The European Economic and Monetary Union is a unique experiment, in which a large group of developed, but structurally heterogeneous economies have created a monetary union. This experiment has serious consequences for the economic dynamics of the countries involved and radically changes the set of possible economic policies. It also poses a challenge to economists, as it requires a reevaluation of the existing knowledge, obtained mostly by studying independent countries.

A prime example of the challenges and issues, which a monetary union can raise, is the recent Eurozone crisis. The events during the crisis raised questions about the differences in debt sustainability between independent countries and members of a monetary union. Policy responses to the crisis involved multilateral sovereign bail outs and discussions about further integration within the monetary union, e.g. in the direction of a banking or fiscal union. On the other hand, the severe social and economic consequences of the crisis in the countries hit the hardest triggered calls for an exit from the monetary union.

Good policy decisions require a sound economic analysis of the issues involved. To prevent further crises in the future we first need to understand the causes and dynamics of the current crisis. This reasoning is the main driving force behind this dissertation, which tries to understand the build-up of macroeconomic imbalances within the union, which contributed to the crisis (in chapter 3), and the contagion dynamics between member countries during the crisis (in chapter 2).

In general, this dissertation contains three essays in international macroeconomics devoted to studying the dynamic behavior of a small open economy within a monetary union. The first two essays explore the role of expectations and informational frictions for a member country of a monetary union; they address the questions of sovereign debt crisis contagion and instability driven by inflation expectations, respectively. The study of issues related to monetary unions and economic crises involves the use of models where non-linearities and uncertainty matter, but those features pose computational challenges to the modeler. Therefore, the last essay proposes a novel method for solving dynamic stochastic models, that preserves the original non-linearity of the model, takes into account uncertainty, but

at the same time allows to approximate the model locally and, hence, avoid the *curse of dimensionality*.

Chapter 2, entitled "Sovereign Default, Exit and Contagion in a Monetary Union" (which is joint work with Sylvester Eijffinger and Burak Uras) deals with the issue of contagion within the European Economic and Monetary Union during the recent crisis. One of the main features of the recent crisis is a simultaneous surge in the cost of borrowing for peripheral EMU countries following the Greek debt-trouble in 2008. In this chapter, I develop a model with optimal default and monetary-union exit decisions of a small open economy. The model can account for the behavior of sovereign bond spreads in the Eurozone with the arrival of the news of Greece potentially exiting the union in the near future. In the theoretical framework, belonging to the monetary-union entails a strong exchange rate peg, which can be abandoned only if the country exits the union. Exit is costly and the cost of exit remains unknown until the first country leaves the union. The theoretical mechanism I explore reveals that while a high expected exit-cost could improve the credibility of a monetary union, uncertainty governing exit-cost realizations could make the monetary-union members prone to surges in interest rates when rumors of a member state exiting arise. I solve the model numerically and quantify that a Grexit-rumors type of shock can triple the default likelihood of an a-priori financially healthy member state. My framework thus provides a novel and quantitatively important explanation for the Eurozone crisis.

Chapter 3, entitled "Unstable Monetary Unions - The Role of Expectations and Past Experience" is complimentary to chapter 2, as it concentrates on the build-up of imbalances within the Eurozone prior to the crisis. This chapter presents a theoretical model that is able to capture the importance of economic experience prior to joining a monetary union for the stability of the country joining. I introduce informational frictions in the form of learning into a model of a monetary union and study how those frictions interact with different economic histories. The model predicts that countries with high inflation experience prior to joining the union accumulate more foreign debt and face a higher risk of economic instability. This suggests that pre-euro heterogeneity in country-specific inflation experience might be a good, and so far neglected, aspiring candidate for a cause behind the imbalances within the Eurozone. I support this claim with an investigation of the empirical patterns of pre-crisis variables in the Eurozone countries. Moreover, the results in this chapter suggest that monetary policy might be not enough to stabilize the economies of member countries within a monetary union, highlighting the importance of complimentary policies.

Chapter 4, entitled "Exact Present Solution - A Gridless Algorithm for Solving Stochastic Dynamic Models", which is (joint work with Wouter den Haan and Pontus Rendahl), is loosely related to the topic of monetary unions, but contributes to the field of international macroeconomics, as it introduces a new numerical method for solving dynamic models, a.o. models of a small open economy. This chapter proposes an algorithm that finds model solutions at a particular point in the state space by solving a simple system of equations. The key step is to characterize future behavior with a Taylor series expansion of the current period's behavior around the contemporaneous values for the state variables. Since current decisions are solved from the original model equations, the solution incorporates nonlinearities and

uncertainty. The algorithm is used to solve the model of a small open economy considered in Coeurdacier et al. (2011), which is a challenging model because it has no steady state and uncertainty is necessary to keep the model well behaved. We show that our algorithm can generate accurate solutions even when the model series are quite volatile. The solutions generated by the risky-steady-state algorithm proposed in Coeurdacier et al. (2011), in contrast, is shown to be not accurate.





# Chapter 2

## Sovereign Default, Exit and Contagion in a Monetary Union

### 2.1 Introduction

1

We develop a model of sovereign debt and default and argue that ex-ante unknown monetary-union exit costs can generate the contagion of a sovereign debt-crisis from a troubled member state (such as Greece) to healthy members of a monetary union (such as Portugal). Our study is motivated with the stylized experience of the southern euro area countries following the Greek debt trouble and the emergence of the rumors concerning the potential of Greece leaving the eurozone (Grexit). The sovereign debt crisis in the euro area is characterized by a simultaneous surge in the cost of borrowing for Southern European governments after 2008. As we document in Figure 1, at the dawn of the crisis in late 2008 the spread on Greek long-term government bonds (relative to the risk-free German bonds) rose from 50 basis points (bps) to 200 bps within a couple of months, and further increased to 1000bps by 2012. Shortly after the outbreak of the Greek debt trouble, the sovereign-bond spreads started to rise in Portugal, Italy and Spain as well. Many argue that this rise in interest rates in Southern Europe was the result of a contagion from Greece.<sup>2</sup> Our dynamic model incorporates a microfounded theory building upon a union-exit cost uncertainty to account for such contagion.

We model widely-accepted characteristics of a monetary-union membership of a small open economy in a dynamic general equilibrium framework. Having committed to an extreme currency-peg through the monetary union membership limits a country's control over its

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<sup>1</sup>This chapter is based on joint work with Sylvester Eijffinger and Burak Uras.

<sup>2</sup>See e.g. Constâncio (2012), Brutti and Sauré (2015) or Favero (2013).

monetary policy and exchange rate, constraining the set of policy instruments available to respond to aggregate shocks. Abandoning a monetary union could especially be attractive if exchange rate misalignments are causing high unemployment rates, as observed in some of the peripheral members of the European Economic and Monetary Union (EMU) since 2009. Despite the high output and unemployment costs suffered during the recent crisis though, we have not observed any departures yet from the EMU. The absence of an exit realization from the EMU could be the result of a high expected cost associated with departing the union.<sup>3</sup>

In our model, we assume that a member state could regain control over its exchange rate policy by leaving the monetary union through incurring a cost of exit. We also assume that this exit cost could be high or low but most importantly the level of it is *ex ante* unknown to the country of our interest - as well as to all other members of the union. There are two ways for the country to uncover the union-exit cost: (i) its government can execute a union-exit itself, and upon completion of this exit, together with the rest of the member states the country learns how costly it is to exit. (ii) It can wait for another member state to exit, such as Greece, and learn from that other member-state's experience how high the cost of union-exit is going to be.

Because of the exit cost uncertainty, the first country exiting the union provides highly valuable information to all other union members. If it turns out that the first exit is a (relative) success, i.e. the exit cost is low, then more countries could follow the path of the first country exiting. In order to replicate the events of the eurozone crisis, during which the Greek debt-trouble spilled over to other EMU countries, we model an exit-rumors shock. In particular, in our quantitative experiments the monetary union gets hit by the exit rumors of a member state, which implies that the uncertainty governing the cost of exiting might resolve over a short period of time. The arrival of exit rumors associated with one member-state (e.g. Greece) impacts the intertemporal debt market interactions of another member state with healthy enough fundamentals (such as Portugal), causing contagion in the form of rising sovereign interest rates also for this country.

The novel qualitative mechanism that we uncover works in the following way: when rumors about Greece exiting emerge, this generates a positive probability that the cost-uncertainty will be resolved soon, with Greece leaving the union. If it turns out that the exit cost is low, soon after Greece's departure Portugal might also consider to leave the union and devalue its newly instated currency - even with sufficiently strong initial fundamentals that prevented Portugal from exiting under the expected (and uncertain) cost of exit state-of-nature.<sup>4</sup> Furthermore, if Portugal would re-denominate and convert its debt into the new currency after its union-exit, then because of the devalued new currency the union-exit also implies a partial default. Therefore, as a result of the convertibility-risk, rational external

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<sup>3</sup>This might be the direct short-term cost in the form of output loss, or a financial turmoil and the operational cost of introducing a new currency, or the long-term cost of foregone international trade facilitated by the pegged currency.

<sup>4</sup>We would like to note that the perceived probability for Portugal to leave the union prior to the Grexit rumors was close to zero, as it was also evident in low interest rates charged by external lenders on Portuguese sovereign bonds before 2009.

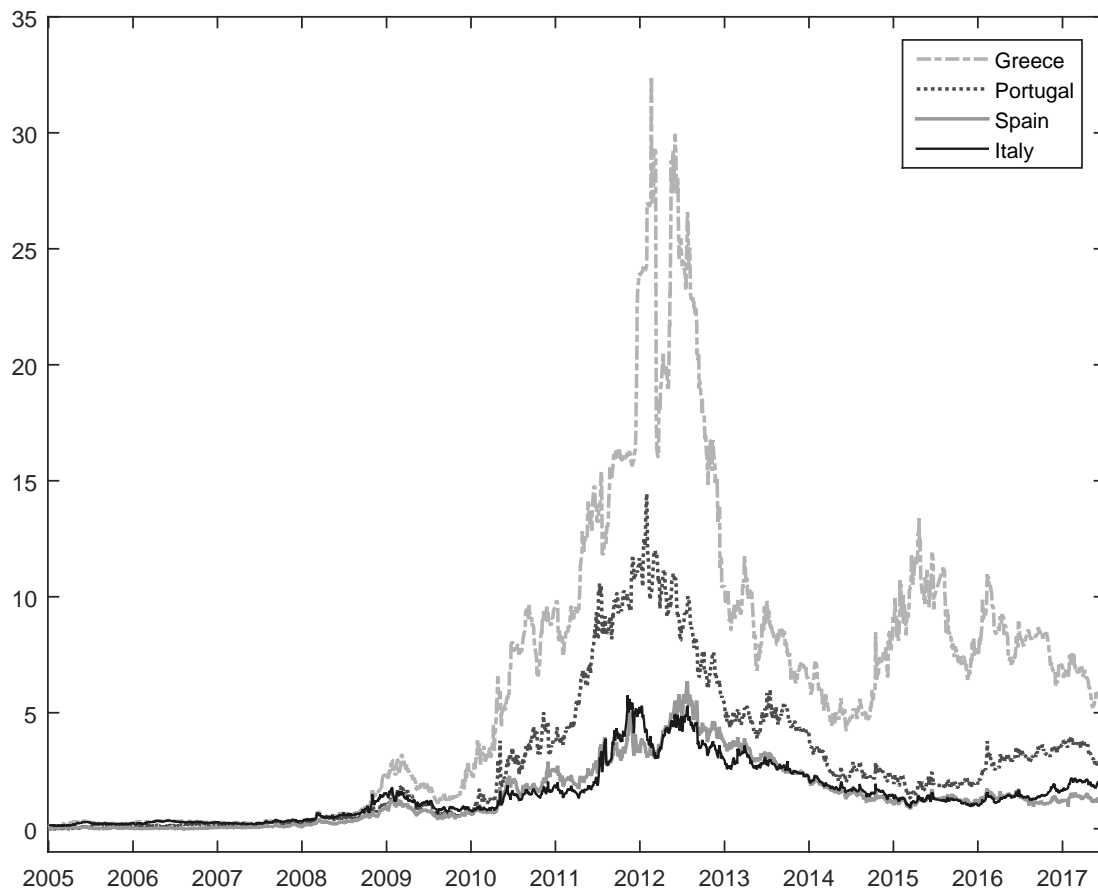


Figure 2.1: Spreads on government bonds of GIPS (relative to German bonds). Daily data on long-term (10-year) yields, obtained from Eurostat.

lenders price the consequences of a potential upcoming exit - and in particular its low-cost realization - and raise interest rates for initially untroubled member states, such as Portugal, following a Grexit-type rumors shock.

The default-premium charged on sovereign borrowing of Portugal would not be so disastrous, if Portugal could easily devalue and relieve the burden of debt. However, until the first exit is completed by Greece, Portugal suffers the cost of a potentially low exit-cost realization without enjoying any of its benefits. In other words, the Portuguese government has to pay a default premium on its bonds resulting from the potential revelation of a low exit-cost in the near future. At the same time though, it still faces the uncertainty about the union-exit cost, such that at an actual exit decision the government has to take the expected union-exit cost as given, under which Portugal might not find it optimal to execute an exit on its own, as observed in reality.

Finally, if a full-fledged default is also available (in the tradition of Eaton and Gersovitz (1981)), this reinforces the rise in interest rates, because high interest rates without the ability to devalue - yet - only worsen the financial and economic conditions for a country

such as Portugal, potentially leading to a full-fledged default.

We introduce the above mechanism into a model of sovereign debt and default that explicitly incorporates a monetary-union exit decision for a small open economy's government. Hereafter we will call the small open economy of our interest as the SOE. In our model default on sovereign debt and exit from the monetary union are two separate but interrelated decisions. A country may default and refuse to pay its external debt, or exit the union, devalue its currency and regain its international competitiveness, or both exit and default simultaneously. Union-exit, through a follow-up currency re-denomination, allows also for a de facto partial default.

We model the punishment for (outright) default as the exclusion from financial markets, accompanied by an output loss. The cost of exiting the monetary union is modeled as a one-time fixed cost, an assumption typical for the literature on currency crises (such as Obstfeld (1994, 1996)). Different from the currency crises literature though we assume that the cost of departure is a priori unknown. Agents form beliefs about the value of the cost of exiting, and the actual value becomes known to all agents only once one of the members completes an exit from the union.<sup>5</sup>

We enrich an Eaton and Gersovitz (1981) type of sovereign default model with our novel monetary union dynamics from the perspective of the SOE. The small open economy that we investigate resembles the key features of Schmitt-Grohé and Uribe (2016) and Na et al. (2018): specifically, (i) the SOE's tradable output is subject to aggregate shocks, (ii) during economic downturns - driven by tradable output shocks - downward rigidity in nominal wages generates involuntary unemployment and a motive for currency devaluation; and, (iii) the government can optimally default on its external debt in order to maximize the aggregate welfare. Default leads to the exclusion of the country from international financial markets and a contraction in tradable output in the future due to financial market exclusion and dead-weight losses.

Utilizing this framework, we investigate the macroeconomic dynamics generated by a news-driven shock associated with the emergence of a member state seeking an opportunity to exit the monetary union, which we interpret as the arrival of the news concerning Grexit rumors. Prior to the rumors shock, the SOE's government takes the expected cost of exit as unknown, forms expectations about it and undertakes exit and default decisions based on that. After exit-rumors the SOE-government undertakes its decisions with the expectation that in the near future the cost of exit could be revealed to all member states. More importantly, also international lenders take this potential short-run information revelation into account and price the bonds of the SOE accordingly. Depending on the initial beliefs about the cost distribution, we show that the rumors shock generates a mechanism capable of worsening the financial conditions for a country with initially good standing, as in some peripheral EMU countries, and push the country into a debt crisis and even to default.

We solve this small open economy model numerically and show that for a relatively

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<sup>5</sup>Instead of assuming that the cost is equal for all countries, one could assume that costs are correlated and the first exit provides partial information about the value of the exit-cost of the remaining countries. This alternative assumption would not change the qualitative implications of the model, but it would complicate the exposition and solution of the model substantially.

moderate expected cost of monetary-union exit, exit-rumors cause rising borrowing spreads and increase the likelihood of default for an initially healthy SOE. This qualitative property turns out to have quantitatively significant implications as well. Specifically, the rumors shock triples the periodic default likelihood of an a-priori healthy SOE, while it raises the periodic default probability by fourfold if the SOE had been experiencing a recession before the exit-rumors shock. The qualitative as well as quantitative properties of our framework are present in a variety of alternative cost specifications that we explore.

The key policy implication from our analysis is that the absence of an explicit exit-clause from a monetary union might be useful to improve the union’s credibility, but it is also a source of financial instability and contagion that policy makers might need to pay attention to.

**Related literature.** We contribute to three strands of literature. Our first contribution is to the large literature on aggregate consequences of currency pegs. In this line of research our work is most related to two recent studies: Na et al. (2018) and Schmitt-Grohé and Uribe (2016). These two papers develop dynamic small open economy models to investigate the welfare cost of currency pegs borne by nominal rigidities in equilibrium wages. On the one hand, Schmitt-Grohé and Uribe (2016) concentrate on the interaction between capital mobility and currency pegs and show that this interaction generates inefficiently high borrowing in international capital markets during booms, which leads to high unemployment during contractions that is driven by rigid wages. The key conclusion from their set-up thus turns out to be the emergence of capital mobility restrictions as an optimal policy instrument in curbing the behavior of nominal wages over the business cycle. On the other hand, Na et al. (2018) study the interactions between default and currency devaluation and illustrate that under rigid wages and fixed exchange rates optimal default takes place when involuntary unemployment is high. Our paper develops a Schmitt-Grohé and Uribe (2016) style small open economy model as well, but differently we investigate the monetary-union dynamics generated by rigid nominal wages.

The second strand of research that we relate to is the literature on endogenous default in the context of sovereign debt markets a la Eaton and Gersovitz (1981). Recent studies that investigated the theoretical features of sovereign default are Aguiar and Gopinath (2006), Arellano (2008), Yue (2010), Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012) and Mendoza and Yue (2012).<sup>6</sup> In this literature attention to the contagion of sovereign default risk has been limited. Two exceptions are the studies by Lizarazo (2013) and Park (2013), both of which explore the role of investors’ attitudes towards charging high risk-premia in sovereign debt markets during times of default and forcing initially untroubled countries into a financial crisis. We contribute to this literature in two ways. First of all, we study a small open economy model in a monetary union and incorporate not only the optimal default decision of the government, but also the optimal union-exit decision. Moreover, we uncover and study a novel theoretical mechanism that generates sovereign debt contagion within a monetary union. The mechanism relies on the potential of information revelation,

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<sup>6</sup>Aguiar and Amador (2014) provide an excellent overview of the recent literature on sovereign debt and default.

in the case when the cost of exiting the union is unknown. The model establishes a link between default and exit decisions and highlights the strong interactions between countries within a monetary union during times of a debt crisis.

Also important for the literature on sovereign debt and default, Durdu et al. (2013) develop a dynamic small open economy model of sovereign debt & default with (noisy) news shocks concerning the next period's TFP realization. This model structure provides the foundation for a theoretical mechanism, through which negative news about next period's TFP raises the default likelihood in the next period and causes a rise in sovereign interest in the current period. Our theoretical model also embeds a similar transmission mechanism. However, different from the set-up of Durdu et al. (2013) we model the monetary-union exit and sovereign default decisions jointly for the government of a small open economy (SOE), using which we then explore the consequences of potential upcoming exit news in the monetary union on the sovereign borrowing costs (and default) for SOE. In this respect, also different from the framework of Durdu et al. (2013) in our set-up it's not negative news per se, but the likelihood of an upcoming information revelation that causes a financial contagion to an initially healthy member of a monetary union.

Finally, and most importantly we contribute to the literature which explores contagion and the dynamics of sovereign bond spreads during the European sovereign debt crisis. Recent empirical studies discuss the puzzling behavior of spreads in the euro-area sovereign bond markets. Bernoth et al. (2012), Aizenman et al. (2013), Beirne and Fratzscher (2013) and Ghosh et al. (2013) using either yield spreads or CDS spreads document that sovereign interest rates were mostly insensitive to fiscal variables prior to the crisis and that this changed drastically during the crisis. Moreover, Ludwig (2014), Kohonen (2014), De Santis (2014), Brutti and Sauré (2015) and Favero (2013) find empirical evidence for contagion in sovereign debt markets within the EMU.<sup>7</sup>

Our paper develops the first theoretical model to analyze contagion within the eurozone through the channel of information revelation.<sup>8</sup> In this respect, our work provides an interpretation for the large body of empirical findings on contagion of sovereign debt crisis in EMU. Our model is also able to explain the findings of Ang and Longstaff (2013) that there is more systemic risk in the eurozone compared to the US. In our framework the systemic risk originates from the shared uncertainty about the union-exit cost and the possibility of an information revelation that is common to all members of the EMU. For the case of the US this systemic channel cannot be operational, because the departure of any individual state from the federation is an extremely unlikely event.

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<sup>7</sup>Beetsma et al. (2013) find also spill-over effects of “news” across troubled countries in the EMU during the crisis, but do not label those as contagion. Similarly, Mink and de Haan (2013) find evidence of a wake-up call among the EMU countries.

<sup>8</sup>Other theoretical models exploring the dynamics of the eurozone crisis include i.a. Aguiar et al. (2015), Corsetti et al. (2014), Corsetti and Dedola (2016), and Broner et al. (2014). These studies do not concentrate on contagion, with the exception of Bolton and Jeanne (2011) who explore contagion through a common lender channel. Alvarez and Dixit (2014) consider the potential of a break-up of the euro-area.

## 2.2 Uncertain Euro-Exit Cost and Domestic-Law Bonds

There are two key features of our model that are important to generate the contagion mechanism. The first one is the uncertainty governing the ex-post revelation of monetary-union exit cost and the other one is the domestic-law bonds which allow debt re-denomination in the case of a domestic currency switch following a monetary-union exit. Both of these features are prevalent characteristics of the EMU.

In the institutional set-up of the EMU there is no explicit legal procedure for abandoning the monetary union. Therefore, until a first-time exit is observed, the member states will naturally not know how painful the process is going to be. In order to highlight an important detail to this end, even the exit protocol for an upcoming potential Grexit that was drawn by teams of Troika after the Greek debt crisis in 2012, had been discussed in absolute secrecy so that premature news & plans would not leak.

The discussions by experts and policy makers following the Greek debt crisis had also proven the existence of a distribution of heterogeneous beliefs regarding the EMU-exit costs and also the belief that the cost of exiting euro is going to be learned by experience. One of the biggest legal and institutional issues, that an exit might trigger, is the uncertainty of whether a country exiting the euro-area would be allowed to remain a member of the EU. The issue arises because the Maastricht Treaty requires all members of the EU to adopt the euro and join the eurozone.<sup>9</sup> The treaty also specifies that the conversion of national currencies is irrevocable and the adoption of the euro irreversible. In a legal analysis of the issue of EU membership after a euro-exit, Athanassiou (2009) concludes that *“a member state’s exit from EMU, without a parallel withdrawal from the EU, would be legally inconceivable.”*

If an EMU exit implied also an EU exit, the whole process would become long and complicated, as it can be currently observed in the example of the UK. The fact that we do not know the legal status of “an EU member exiting the EMU” adds a very significant component to the uncertainty governing the union-exit cost.

On the high-cost expectations side, it had been highlighted that the short-term effects of Grexit would be so disruptive that it could lead to a civil unrest and cause a very significant contraction in consumption and wealth over a long horizon. On the low-cost side, proponents had been arguing that re-introducing drachma would be easy enough such that in the short-run exports and tourism can boost quickly to overcome the cost of abandoning the euro - allowing Greece to recover fast. To give a particular example from this end, in a column on May 2015, Paul Krugman stated the following:

*“[T]he bigger question is what happens a year or two after Grexit, where the real risk to the euro is not that Greece will fail but that it will succeed.”* (New York Times. May 25, 2015)

Basically, a successful Grexit in the near future could trigger a domino effect of other successful EMU-exit experiences. Moreover, if Grexit would turn out to be a success, the

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<sup>9</sup>The only two exceptions are the UK and Denmark, who have negotiated opt-out clauses. The remaining countries are required to join, even though a formal deadline has not been set.



expertise developed in Greece could easily be hired in other EMU countries, which might be interested in an easy-way-out from the eurozone as well.

The realistic existence of an exit-cost uncertainty and in particular the possibility of further successful departures in the near future following a low-cost (successful) realization of Grexit are what we formalize and study in this paper.

Another important aspect of our theoretical set-up is the association of a monetary-union exit with an endogenous currency devaluation and debt re-denomination. This feature of the model is highly prevalent for the context of the EMU as well. Specifically, as documented by Schumacher et al. (2015) between 2003-2014 on average 90% of the sovereign bonds originated in Portugal and in Spain and 99% of sovereign bonds issued in Italy were issued under the domestic law. Domestic law bonds allow a sovereign to change the denomination of its external debt if the domestic currency of the country would change.

The existence of this option for a sovereign government implies for external lenders that the value of outstanding sovereign debt could contract after a currency transition, such as abandoning the euro. In particular for the eurozone countries, because of the high degree of exchange rate misalignments, the main rationale for abandoning the monetary union is the possibility to introduce a new currency and devalue. In this respect, the risk of re-denomination is not only a theoretical possibility in EMU. This convertibility-risk has been highlighted even by the President of the European Central Bank, Mario Draghi, during the eurozone debt crisis:

*“Then there’s another dimension to this that has to do with the premia that are being charged on sovereign states’ borrowings. These premia have to do, as I said, with default, with liquidity, but they also have to do more and more with convertibility, with the risk of convertibility.”* (London, July 26, 2012, source: ECB (2012))

The words of Mario Draghi are empirically confirmed by De Santis (2015), who proposes to measure the convertibility risk as the spread between euro- and dollar-denominated sovereign bonds. Furthermore, to control for the differences in the liquidity premia in those two markets, they take the difference between this measure for a risky country and a safe country, e.g. the difference between the Spanish and the German spreads. He documents the existence of a convertibility risk premium for Spain, Italy and France during the period of 2011-2013. This suggests that in this time period markets were taking into account the risk of an EMU exit and a consequent re-denomination of sovereign bonds in those countries. de Haan et al. (2014) control for re-denomination risk by time-varying parameters. Using an alternative approach, Kriwoluzky et al. (2015) estimate a DSGE model with exogenous exit expectations for Greece and find a significant contribution of these expectations to Greek risk premia and debt dynamics. Their results imply that exit expectations might drive a country into a debt crisis, which is consistent with the mechanism that we present in this paper.

Thus, the legal framework in Europe permits countries to re-denominate their debt after a monetary-union exit and anecdotal and empirical evidence points that this risk had been

priced in by investors during the eurozone debt crisis - supporting one of the key features making up the backbone of our framework.

## 2.3 Model

We investigate the dynamic behavior of a small open economy, that we call the SOE, in a monetary union. The model builds upon the structure developed by Na et al. (2018) which is suitable to investigate the interactions between currency devaluation and sovereign-debt default in the tradition of Eaton and Gersovitz (1981). We enrich the framework of Na et al. (2018) by incorporating a monetary-union exit decision for the SOE. Monetary union members share a common currency, whose nominal exchange rate is fixed at an exogenously specified policy-rate. A member state, such as the SOE, can exit the union in any time-period and adopt its own domestic currency. If adopted, the country's own currency allows the government of the SOE to choose its own devaluation policy. As in Na et al. (2018) and Schmitt-Grohé and Uribe (2016) devaluation is desirable during times of an economic downturn because of the presence of a downward rigidity in nominal wages. Importantly, in our framework devaluation also reduces the burden of debt issued under the domestic law, as the country is allowed to convert the debt from the currency of the union into its own domestic currency upon monetary-union exit.

A key feature of the model is the costly exit from the monetary union. Specifically, in order to exit the monetary union and switch to its own domestic currency, the SOE has to incur a one-time cost. This cost is similar to the cost of abandoning an exchange rate peg, as traditionally assumed in the currency crises literature. As a crucial difference from the past literature, the level of the union-exit cost is uncertain and is revealed only when a member state completes an exit from the union. Given a set of initial conditions - which resemble the situation of the EMU at the on-set of the Greek sovereign-debt crisis and the emergence of Grexit rumors, we will show that the exit-cost uncertainty is capable of generating a mechanism for contagion of a sovereign debt crisis in the monetary union. Before we move on describing the key mechanism of the model, at first we present the decision programs of households, firms and the government.<sup>10</sup>

### 2.3.1 Households

There is a large number of households whose preferences over consumption goods are described as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (2.1)$$

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<sup>10</sup>The definition of the recursive equilibrium and the timing of the model can be found in Appendix B and Appendix C, respectively.

where  $c_t$  is consumption. The period utility function  $U$  is strictly increasing and strictly concave. The parameter  $\beta$  denotes the discount factor, with  $0 < \beta < 1$ , and  $\mathbb{E}_0$  is the expectation operator. The consumption good is an aggregator of tradable consumption,  $c_t^T$ , and non-tradable consumption,  $c_t^N$ . The aggregation technology exhibits constant-elasticity-of-substitution and it is specified as

$$c_t = \left( a(c_t^T)^{\frac{\varepsilon-1}{\varepsilon}} + (1-a)(c_t^N)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \text{ with } \varepsilon > 1. \quad (2.2)$$

Households do not have direct access to international financial markets, but they receive transfers  $T_t$  from the government which borrows and saves on their behalf in international financial markets.<sup>11</sup> The budget constraint of each household is expressed as

$$P_t^T c_t^T + P_t^N c_t^N = P_t^T \tilde{y}_t^T + W_t l_t + T_t + \Phi_t. \quad (2.3)$$

At households' budget constraint  $P_t^T$  and  $P_t^N$  denote the nominal prices of the tradable- and non-tradable goods respectively. We assume that the households' endowment of tradable goods,  $\tilde{y}_t^T$ , follows an exogenously determined stochastic process, that is taken as given by every household. The variable  $W_t$  is the nominal wage rate earned from providing labor services in the non-tradable good sector. The variable  $l_t$  is the hours worked by a household. Finally,  $\Phi_t$  is the nominal profits received from the ownership of firms which produce the non-tradable good.

Households inelastically supply  $\bar{l}$  hours to the labor market, but they may not be able to sell every labor-hour that they are endowed with: the model generates involuntary unemployment in equilibrium whenever  $W_t$  is too high. This nominal wage rigidity is the key for the dynamic behavior of the economy, which gives rise to the following constraint

$$l_t \leq \bar{l}. \quad (2.4)$$

Households take  $P_t^T, P_t^N, W_t, l_t, \Phi_t, T_t$  as given and maximize (2.1) subject to (2.2), (2.3), (2.4), and the exogenous output process for tradables - to be specified below - by choosing contingent plans  $\{c_t^T, c_t^N\}$ . The optimality condition for tradable and non-tradable good consumption gives

$$\mathcal{P}_t = \frac{1-a}{a} \left( \frac{c_t^T}{c_t^N} \right)^{\frac{1}{\varepsilon}}, \quad (2.5)$$

where  $\mathcal{P}_t \equiv P_t^N / P_t^T$  is the price of non-tradable goods relative to tradable goods.

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<sup>11</sup>The transfers could also be negative, in which case they constitute lump-sum taxes.

### 2.3.2 Firms

Non-tradable output of the SOE is produced by perfectly competitive firms. Each firm operates a production technology specified as

$$y_t^N = F(l_t), \quad (2.6)$$

where  $F(\cdot)$  is strictly increasing and strictly concave. Firms demand labor hours from households to maximize profits given by

$$\Phi_t = P_t^N F(l_t) - W_t l_t. \quad (2.7)$$

The optimality condition associated with firms' maximization problem yields  $P_t^N F'(l_t) = W_t$ . Dividing both sides of this expression by  $P_t^T$  gives

$$\mathcal{P}_t F'(l_t) = w_t, \quad (2.8)$$

with  $w_t \equiv W_t/P_t^T$  denoting the real wage denominated in terms of tradables.

### Downward Nominal Rigidity

Following Na et al. (2018) and Schmitt-Grohé and Uribe (2016) we assume that wages are downwardly rigid. Specifically, there is a lower bound on the growth rate of equilibrium nominal wages such as

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \quad (2.9)$$

The parameter  $\gamma$  captures the degree of downward nominal wage rigidity. The higher is  $\gamma$ , the more rigid are the nominal wages. As also argued by Schmitt-Grohé and Uribe (2016), downward wage rigidity is a stylized empirical fact especially for the case of the European Economic and Monetary Union: in early 2000s euro-area countries experienced substantial appreciations in hourly wages, caused mostly by large increases in capital inflows. Following the drying up of capital inflows at the onset of the 2007/2008 global financial crisis, aggregate demand collapsed. However, hourly wages in the post-2008 era remained at the peak-level that they achieved before 2008. The combination of falling demand and rigid wages, together with the absence of local currencies that can be depreciated during the downturn, led to massive increases in involuntary unemployment throughout the eurozone, especially in peripheral countries.

The presence of downward rigidity in nominal wages gives rise to involuntary unemployment in our model as in Na et al. (2018) and Schmitt-Grohé and Uribe (2016), such that  $\bar{l} - l_t > 0$  is a frequent feature of the economy whose dynamic implications we investigate. Specifically, nominal rigidities imply a complementary slackness condition in the form of

$(\bar{l} - l_t)(W_t - \gamma W_{t-1}) = 0$ , which could be also expressed as

$$(\bar{l} - l_t) \left( w_t - \gamma w_{t-1} \frac{P_{t-1}^T}{P_t^T} \right) = 0. \quad (2.10)$$

The condition (2.10) implies that periods of unemployment are always accompanied with a binding nominal wage constraint.

## Partial Equilibrium in Labor and Goods Markets

The first requirement of the competitive equilibrium is that the market for non-traded goods clears in all periods, such that

$$c_t^N = y_t^N \quad (2.11)$$

for all  $t$ .

We denote the foreign price of tradables with  $P_t^*$  and assume that the law of one price holds for tradables

$$P_t^T = P_t^* \tilde{\epsilon}_t, \quad (2.12)$$

where  $\tilde{\epsilon}_t$  is the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. As long as the country is a member of the monetary union, the nominal exchange rate is simply given by  $\tilde{\epsilon}_t = 1$ . Furthermore, we assume that the foreign price of tradables is fixed and set at  $P_t^* = 1$  such that

$$P_t^T = \begin{cases} 1, & \text{if the country remains in the union,} \\ \epsilon_t, & \text{if the country is outside the union,} \end{cases} \quad (2.13)$$

where  $\epsilon_t$  is to be determined at the discretion of the domestic government following upon a potential exit of the SOE from the monetary union. Plugging the above into equation (2.10) yields a modified slackness condition

$$(\bar{l} - l_t) \left( w_t - \gamma w_{t-1} \frac{\tilde{\epsilon}_{t-1}}{\tilde{\epsilon}_t} \right) = 0. \quad (2.14)$$

The partial competitive equilibrium in labor and goods markets is a result of firms and households making optimal decisions and interacting by taking the tradable endowment process and the policies of the government as given. We define the partial competitive equilibrium as follows.

**Definition (Partial Competitive Equilibrium).** A partial competitive equilibrium is a set of stochastic processes  $\{c_t^T, c_t^N, c_t, l_t, W_t, P_t^N, P_t^T\}$  satisfying (2.2), (2.3), (2.4), (2.5), (2.6),

(2.8), (2.9), (2.11), (2.14), given the processes  $\{\tilde{y}_t^T, \tilde{\epsilon}_t, T_t\}$  and the initial condition  $w_0$ .

### 2.3.3 Government

The key economic actor in the model is the government of the SOE. In every period, the government decides on the external borrowing of the country in international financial markets and also whether to default on its outstanding external debt. The government also decides whether to retain the membership of the SOE in a monetary union - governed by a fixed exchange rate regime - and if it decides to exit the union, it also chooses the follow-up exchange rate policy of the country. At first we present the possible regimes that the SOE can start any time-period with, depending on the government's past external-debt-default and union-exit decisions, and then delineate the decision processes of the government that lead to these regimes.

At the beginning of a period  $t$  the country may be in one of four possible regimes. The SOE can be in the monetary union while being either in *good financial standing* - as of the beginning of the period  $t$ , or while being in the *default-status* if the government reneged on its external debt at some point in time before period  $t$ . The SOE might have also exited the monetary union before the time period  $t$  and have the *exit* status in period  $t$  either while having a good financial standing or while being in the default status.

The full set of possible transitions between different regimes is presented graphically in Figure 2.2. The SOE in the UNION regime is a member of the monetary union with full access to international financial markets. While being in the UNION regime the government can retain the country in this regime by keeping membership in the currency union and at the same time continue to honor the country's external debt obligations. The government can also move the SOE into one of the three remaining regimes by fully defaulting on its entire outstanding debt (the regime we denote as *DEFAULT*), by exiting the union (the regime denoted as *EXIT*) or by defaulting fully and at the same exiting the monetary union (the regime denoted as *AUTARKY*).<sup>12</sup> We turn next to delineating the decision processes of the government and their economic implications that give rise to these four possible regimes.

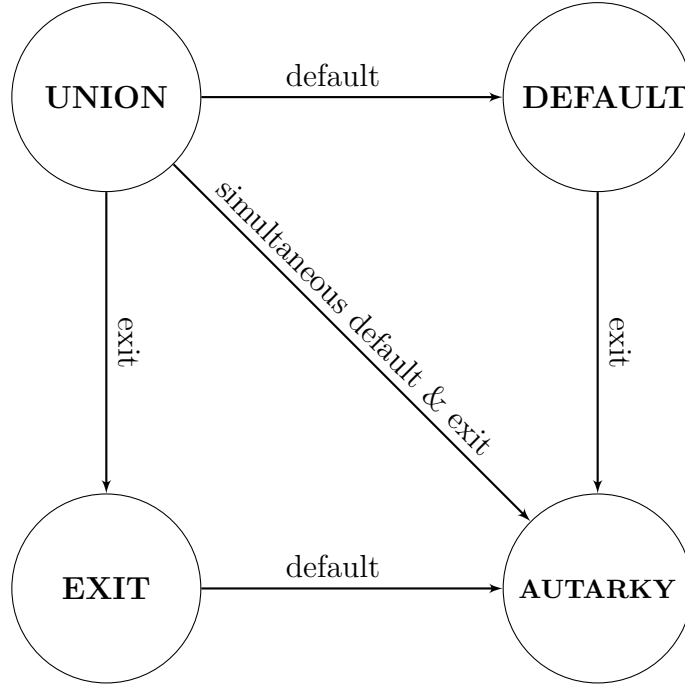
### Government's International Financial Market Policies

As long as the government of the SOE is in good financial standing - such that a default on its external debt had never been executed before, it can issue one-period, non-state contingent bonds and raise funds in international markets. The bonds are sold at the nominal price  $q_t$ , denominated in terms of the domestic currency of the country. This means that for the case of a union-member the external debt is denominated in terms of the union-currency

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<sup>12</sup>For tractability we omit the possibility that once a country defaults it may reenter the international financial markets. The reentry assumption is standard in the literature and allows to match better the moments observed in the data, but does not qualitatively change the properties of our model. The Bellman equations and the scheme of regime switches for the case with reentry can be found in the Technical Appendix available from the corresponding author upon request.

Figure 2.2: Possible regime switches in the economy



whereas for a country outside the union debt is denominated in terms of the country's own domestic currency. The legal framework (as in the case of eurozone) allows the government to switch the denomination of the SOE's debt from the union-currency to the SOE's own currency following upon an exit from the union. The face value of the government bond,  $d_{t+1}$ , specifies the value that needs to be repaid in the next period. The government uses the funds raised in international financial markets to provide transfers to the households ( $T_t$ ). The intertemporal budget constraint of the government is expressed as

$$T_t = (q_t d_{t+1} - d_t)(1 - \mathcal{D}_t), \quad (2.15)$$

where  $\mathcal{D}_t$  is the default history up to (and including) period  $t$ , where  $\mathcal{D}_t = 0$  indicates no default up until period  $t$ , and  $\mathcal{D}_t$  takes the value 1 if a default has taken place in period  $t$  or in any of the preceding periods. We distinguish between the default history  $\mathcal{D}_t$  and the default decision  $D_t$ . The latter takes on the value  $D_t = 1$  only in the period of default and 0 in all remaining periods, and the history takes the value 1 in all periods starting from the default period.<sup>13</sup>

If the government decides to default on its external debt ( $D_t = 1$ ) in a period  $t$ , in that

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<sup>13</sup>The relationship between  $\mathcal{D}_t$  and  $D_t$  can be described by  $1 - \mathcal{D}_t = \prod_{i=0}^t (1 - D_i)$ , i.e. the default history  $\mathcal{D}_t$  takes on the value 0 if and only if the default decision was 0 (i.e. repay) in all periods from 0 up to  $t$ .

particular period the entire debt repayment obligations of SOE to the foreign lenders do not get honored. Following the incidence of a default in  $t$ , in the same period  $t$  the government loses its access to international financial markets and this exclusion remains effective forever. As standard in the literature on sovereign debt and default, we assume that in any time-period after SOE switches to the bad financial standing ( $\mathcal{D}_t = 1$ ), it suffers an output loss worth of  $L(y_t^T)$  with  $L(\cdot) \geq 0$  and  $L'(\cdot) \geq 0$ .<sup>14</sup> This means that the flow of tradables available to households is equal to

$$\tilde{y}_t^T = y_t^T - \mathcal{D}_t L(y_t^T), \quad (2.16)$$

where the basic endowment  $y_t^T$  follows an AR(1) process

$$\ln(y_t^T) = \rho \ln(y_{t-1}^T) + (1 - \rho) \ln(y^T) + \mu_t, \quad (2.17)$$

with  $y^T$  denoting the steady-state level of tradable output.

## Government's Nominal Exchange Rate Policies

The SOE starts out as a member of the monetary union. This means that the SOE initially operates under an extreme version of an exchange rate peg: it uses the currency of the monetary union as its domestic currency, which implies an exchange rate fixed at  $\tilde{\epsilon} = 1$ . The only way for the government to deviate from this exchange rate is to exit the union and introduce its own domestic currency. As long as the SOE is a member of the union, the government undertakes a decision at the beginning of every period whether to remain as a member state in that particular period or to exit the union and set the exchange rate of the country equal to  $\epsilon_t$  at its own discretion. The government's "remain-or-exit decision" is a discrete choice denoted by  $X_t$ , with  $X_t = 0$  indicating "to remain" in the union in period  $t$  and  $X_t = 1$  indicating "to exit" from the union in period  $t$  in order to introduce SOE's own domestic currency as of period  $t$ . We assume that once the SOE exits the union it cannot reenter.<sup>15</sup>

Next to the exit decision  $X_t$ , we introduce also a variable representing the exit history of the SOE  $\mathcal{X}_t$ . We use  $\mathcal{X}_t = 0$  to indicate a country that has never exited and thus remains a member state of the union, and  $\mathcal{X}_t = 1$  to indicate a country outside of the union, i.e. a country that has executed an exit in period  $t$  or in any preceding period.

As an important feature of the model we assume costly union-exit. Costly exit means that abandoning the currency of the union as the domestic-currency of the SOE is associated with a one-time loss of  $\tilde{C}$  units of utility in the period of exit. The one-time utility loss associated with exiting the monetary union is additive and therefore it does not interact with the utility from consuming tradable and non-tradable goods. One can easily motivate this monetary-

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<sup>14</sup>Mendoza and Yue (2012) provide a theoretical microfoundation for the output loss after default and document its empirical validity.

<sup>15</sup>This assumption helps with tractability but does not affect our qualitative or quantitative findings. It is unclear whether a country exiting the EMU would be allowed to rejoin in the future.



union exit cost, because it requires time and effort in order to legally abandon a currency and switch to another one by replacing the old one at all transactions. It is also standard in the currency crises literature to assume that abandoning a currency-peg is a costly decision, where as in our framework in some studies the cost of abandoning the peg is incorporated as a utility loss.<sup>16</sup>

What distinguishes the monetary union exit cost from the cost of abandoning a standard currency-peg is that the former is expected to be governed by a large uncertainty because of the necessity to literally replace the currency used in transactions, which - as a key and novel feature - we also incorporate into our model.

The motivation for the cost-uncertainty can be twofold: First, as in the case of the euro area, to improve the credibility of the union, the founders might have decided not to include any explicit legal exit-clause, making any potential exit uncertain and changing the unilateral decision of currency abandonment into a multilateral negotiation process between the country exiting and the remaining members of the union. Second, since no developed country exited a monetary union in modern times (as is also the case for the euro area), there is no past experience that a decision-maker government could exploit to precisely estimate the cost of the union-exit.<sup>17</sup>

How difficult the implementation of an exit is going to be, gets understood ex-post - only upon the completion of a de-facto exit from the union. Therefore, the first exit from the union provides a valuable case study for other member states which might consider to exit in the future. In order to capture this important aspect of monetary-union membership, we assume that the utility cost of exit is uncertain until the first-exit. After the completion of the first-time exit, the cost figure gets revealed to all member states and remains at that level forever. This means that if the government of the SOE whose behavior we investigate wants to implement a first-time exit from the union, it has to form beliefs about  $\tilde{C}$ . The beliefs about the exit cost are given by a distribution function  $G(C)$  and they are shared by all economic actors of the model. There are two exogenous shocks at the union level concerning the revelation of the true  $\tilde{C}$ , which influence economic decisions and outcomes for the SOE and importantly also for its external lenders. We formalize them as follows.

We describe the state of the monetary union in any time period  $t$  from the perspective of the SOE, by excluding the actions of the SOE and their implications on the rest of the monetary union. This is how we isolate and study the effects of “exogenous shocks” stemming from the monetary-union on the macroeconomic dynamics of the SOE.

The overall state of the monetary union, from the SOE’s perspective, as of the beginning of any time period  $t$  is described by the vector  $\mathcal{M}_t$ . The state of the union  $\mathcal{M}_t$  is an information-set containing the past-history of “exits” from the union until period  $t$  (denoted

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<sup>16</sup>Obstfeld (1994, 1996, 1997) are prominent examples of currency crises models where the cost of abandoning the peg is introduced as an additive utility term.

<sup>17</sup>The uncertainty governing the economic and political costs of Brexit may serve as an example of how difficult it is to predict the consequences of an exit from any union, if it is unprecedented. This is despite the fact that there is a legal clause for exiting the EU as compared to the lack of a clause for the EMU-exit. Another argument for the uncertainty of the exit cost in the case of the EMU is the fact that it is unsure whether a country exiting the EMU would be also forced to exit the EU.

with  $h_t$ ) and the current-period news associated with the existence of a member state seeking an option to exit, which we call as “exit rumors” (denoted with  $e_t$ ). In this respect,  $\mathcal{M}_t = (h_t, e_t)$ .

To the end of exit-histories, there are two potential histories relevant for the SOE: the existence of at least one member state - other than the SOE - that departed from the union before period  $t$  (a state of the history which we denote with  $x$ ); and, the absence of any exit until period  $t$  (a state of history denoted by  $u$ ). Hence,  $h_t \in \mathcal{H}_t \equiv \{x, u\}$  for all  $t$ . Exit shocks get realized as of the end of each period. This means the exit of a member state which affects the relevant monetary-union history for the SOE in period  $t$  gets realized at the end of period  $t - 1$ .

With respect to the exit-rumors stemming from one of the other members of the union in period  $t$ , there are also two relevant states for the SOE: the existence of at least one member state - again other than the SOE - considering an exit in period  $t$  (denoted with the state  $s$ ) and the absence of a member state seeking an exit (denoted with the state  $n$ ). Therefore,  $e_t \in \mathcal{E}_t \equiv \{s, n\}$  for all  $t$ .

Next we specify the transition of the realized states in period  $t$ ,  $\mathcal{M}_t = (h_t, e_t)$ , into the future states of period  $t + 1$ ,  $\mathcal{M}_{t+1} = (h_{t+1}, e_{t+1})$ . We first note that  $x$  is an absorbing state for the case of historical transitions, i.e.  $\text{prob}(h_{t+1} = x | h_t = x) = 1$  for any  $e_t \in \{s, n\}$ . This means that once a first-exit from the union is realized, the arrival of exit rumors after that first-exit become inconsequential for union-wide economic outcomes. The likelihood of transitioning from history  $h_t = u$  to history  $h_{t+1} = x$  depends though on the existence of exit rumors in period  $t$ . To this end, we assume that

$$\text{prob}(h_{t+1} = x | h_t = u, e_t = s) = p > \text{prob}(h_{t+1} = x | h_t = u, e_t = n) = 0,$$

which implies that if there are exit-rumors about at least one member-state’s potential departure in period  $t$ , the (first) actual exit from the union will materialize as of the end of period  $t$  with probability  $p$ . If there are no exit-rumors in the union (excluding any exit-intentions that the SOE might have), no exit would materialize in the same period, and hence in this case the union would remain into the next period as a whole as long as the SOE does not execute an exit on its own. Therefore, superscripting the next period states with primes, the probability transition matrix for the state of histories of the monetary-union that the SOE will take as given ( $\Omega(h' = x | \mathcal{H}, \mathcal{E})$ ) is expressed as

$$\Omega(h' = x | \mathcal{H}, \mathcal{E}) = \begin{bmatrix} \text{prob}(x|x, s) & \text{prob}(x|x, n) \\ \text{prob}(x|u, s) & \text{prob}(x|u, n) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ p & 0 \end{bmatrix}. \quad (2.18)$$

With respect to the transition of exit-rumors from one period to the next, we assume that

$prob(e_{t+1} = s|e_t = s) = \tilde{p}_s$  and  $prob(e_{t+1} = s|e_t = n) = \tilde{p}_n$  for all  $h \in \{x, u\}$ . Hence:

$$\Lambda(e' = s|\mathcal{H}, \mathcal{E}) = \begin{bmatrix} prob(s|x, s) & prob(s|x, n) \\ prob(s|u, s) & prob(s|u, n) \end{bmatrix} = \begin{bmatrix} \tilde{p}_s & \tilde{p}_n \\ \tilde{p}_s & \tilde{p}_n \end{bmatrix}, \quad (2.19)$$

with  $\tilde{p}_s > \tilde{p}_n$ . The exit-rumor shocks and their consequences for an actual exit at the union-level imply the potential of the revelation of the actual exit cost in the short-run, which has important theoretical and quantitative effects, as we will discuss below.

If the government of the SOE exits the union by taking the available exit-cost figures as given, effectively in the same period of the exit it introduces its own domestic currency. In this case it may also find it optimal to devalue the new currency against the currency of the union in order to relax the burden associated with a binding nominal-wage rigidity constraint. As it is apparent from equation (2.14), whenever the wage part of the slackness condition is binding, the government may eliminate involuntary unemployment by relaxing the constraint with a devaluation. The optimal devaluation strategy for a small open economy with nominal wage rigidities is discussed extensively in Schmitt-Grohé and Uribe (2016) and Na et al. (2018), in which the authors show that devaluations are desirable during economic contractions and default episodes.

In our framework, a devaluation has a second role. Since the government of the SOE issues bonds under the domestic law, the denomination of the external debt may be converted into the domestic currency upon exiting the monetary union. If the union-exit is followed by a devaluation and debt conversion, the exit is then equivalent to a partial default. This is costly for external lenders, because the value of debt remains constant in the newly introduced local currency, but the currency itself loses value as expressed in tradables or the union's currency.<sup>18</sup> This partial default of the government through devaluation and debt-conversion can be executed only once, only in the period of the SOE's exit from the monetary union. In any time-period following the period of the exit the SOE issues inflation-indexed bonds, which is equivalent to the bonds being denominated in tradables or a foreign currency.

Na et al. (2018) show also that a decision-maker government is indifferent between any devaluation that is larger than the minimal devaluation guaranteeing full employment. Since in our model the devaluation has the additional partial default effect, the government would always choose an infinite devaluation to wipe away all debt. To prevent this we assume that the government is limited to choosing the minimal devaluation *a la* Na et al. (2018). This assumption is made for simplicity and transparency, as alternatively we could assume an exit cost that is dependent on the size of the devaluation.

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<sup>18</sup>Schumacher et al. (2015) show that most government bonds in the eurozone were issued under domestic law, and that the risk of a conversion was priced in by international investors during the crisis. They also document that the crisis-hit-countries have used foreign law issuance after the crisis to reenter credit markets.

## Value Functions and Government's Optimization Program

Let us denote with  $\mathcal{S}_t$  the state of the SOE in period  $t$ . The state of the country encompasses the exogenous endowment process  $\tilde{y}_t^T$ , the past equilibrium wage rates, as well as the past debt, exchange rate, default and exit decisions of the government, so that  $\mathcal{S}_t = \{\tilde{y}_t^T, W_{t-1}, d_t, \epsilon_{t-1}, \mathcal{D}_{t-1}, \mathcal{X}_{t-1}\}$ . The government's objective is to maximize the households' expected lifetime utility by taking  $\mathcal{S}_t$  and the state of the monetary union,  $\mathcal{M}_t$ , and the conditions described in the definition of partial equilibrium as given. As delineated above, the policy instruments of the government are fourfold: (i) the government decides on whether to keep the country in the monetary union and (ii) following upon an exit the nominal exchange rate of its newly introduced currency. (iii) The government also undertakes a decision on whether to default on the country's external debt and (iv) in any time-period of good financial standing it chooses the level of external debt for the next period.

The value function for an SOE in the UNION regime at the beginning of period  $t$  is

$$\begin{aligned} V^U(\mathcal{S}_t; \mathcal{M}_t) = & \max_{c_t, d_{t+1}, D_t, X_t} \left\{ u(c_t) + (1 - D_t)(1 - X_t)\beta\mathbb{E}_t [V^U(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] \right. \\ & + (1 - D_t)X_t \left( \beta\mathbb{E}_t [V^X(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] - \tilde{C} \right) \\ & + D_t(1 - X_t)\beta\mathbb{E}_t [V^D(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] \\ & \left. + D_tX_t \left( \beta\mathbb{E}_t [V^A(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] - \tilde{C} \right) \right\}, \end{aligned} \quad (2.20)$$

subject to (2.3) and (2.15). We note that the value associated with being a member of the union ( $V^U(\mathcal{S}_t; \mathcal{M}_t)$ ) in period  $t$  takes into account the possibility of leaving the union in the same time period by incurring the one-time (additive) utility loss of  $\tilde{C}$ . We also highlight that since the utility loss from exiting the union is additive, it does not interact with the utility from consumption in the expression of the value function. Furthermore,  $V^X$  is the value function for an SOE in the EXIT regime at the beginning of period  $t$ , which can be represented as

$$\begin{aligned} V^X(\mathcal{S}_t; \mathcal{M}_t) = & \max_{c_t, d_{t+1}, \epsilon_t, D_t} \left\{ u(c_t) + (1 - D_t)\beta\mathbb{E}_t [V^X(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] \right. \\ & \left. + D_t\beta\mathbb{E}_t [V^A(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] \right\}, \end{aligned} \quad (2.21)$$

subject to (2.3) and (2.15).  $V^D$  is the value function for an SOE in the DEFAULT regime at the beginning of period  $t$

$$\begin{aligned} V^D(\mathcal{S}_t; \mathcal{M}_t) = & \max_{c_t, X_t} \left\{ u(c_t) + (1 - X_t)\beta\mathbb{E}_t [V^D(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] \right. \\ & \left. + X_t \left( \beta\mathbb{E}_t [V^A(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})] - \tilde{C} \right) \right\}, \end{aligned} \quad (2.22)$$

subject to (2.3) and (2.15), where  $V^D(\mathcal{S}_t; \mathcal{M}_t)$  in period  $t$  takes into account the possibility of incurring the utility loss associated with the monetary union exit in the same time period. Finally,  $V^A$  is the value function for an SOE in the AUTARKY regime at the beginning of period  $t$

$$V^A(\mathcal{S}_t; \mathcal{M}_t) = \max_{c_t, \epsilon_t} \{u(c_t) + \beta \mathbb{E}_t [V^A(\mathcal{S}_{t+1}; \mathcal{M}_{t+1})]\}. \quad (2.23)$$

subject to (2.3) and (2.15).

### 2.3.4 External Lenders

The bonds issued by the government of the SOE are traded in international financial markets. The external (foreign) lenders buying these bonds are assumed to be risk neutral. With  $r^*$  denoting the risk free interest rate, the no-arbitrage condition for sovereign bonds under the UNION regime takes the form of

$$1 + r^* = \frac{1}{q_t} \mathbb{E}_t \left[ (1 - D_{t+1}) \left( 1 - X_{t+1} \frac{\tilde{\epsilon}_t}{\tilde{\epsilon}_{t+1}} \right) | \mathcal{D}_t = 0, \mathcal{X}_t = 0 \right]. \quad (2.24)$$

This condition states that external lenders demand a premium for the possibility of an outright default as well as for the possibility of a partial default through debt-conversion. Put differently, the price of the SOE-bonds depends on the probability of the SOE remaining in the UNION regime. As also delineated in the government's program, the partial-default channel that raises the cost of borrowing in international financial markets is novel for this class of models. Finally, the expectation formation of external lenders is key for our analysis, which among other things is also conditional on shocks to tradables and importantly on shocks to the state of the union concerning the revelation of the union-exit cost.

A similar no-arbitrage condition for government bonds holds for a country in the EXIT regime

$$1 + r^* = \frac{1}{q_t} \mathbb{E}_t [1 - D_{t+1} | \mathcal{D}_t = 0, \mathcal{X}_t = 1], \quad (2.25)$$

where under the EXIT regime the bonds are inflation-indexed and thus investors do not face any exchange rate risk.

### 2.3.5 Qualitative Properties of the Model

The probability of transitioning from history- $u$  (no past exits) to history- $x$  (with exits) at the level of the union in-between any two periods and the dependence of this transition probability on exit-rumors affect the decisions of the SOE that are intertemporally relevant, such as the borrowing of the SOE's government in international financial markets. The key mechanism that gives rise to this important property is related to the "potential resolution" of the cost-uncertainty in the short-run. We illustrate the mechanism as follows.

Let us consider the following distribution function of ex-ante beliefs associated with the union-exit utility loss,  $G(C)$ , that is shared by the SOE's government and the external lenders

$$\tilde{C} = \begin{cases} C^L, & w/prob. \zeta, \\ C^H, & w/prob. 1 - \zeta, \end{cases} \quad (2.26)$$

with  $C^L < C^H$  and an implied expected value of  $C^e = \zeta C^L + (1 - \zeta)C^H$ . Furthermore, without loss of generality let us also assume that the exit rumors shock does not exhibit time-series persistence (i.e.  $\tilde{p}_s = 0$ ).<sup>19</sup>

Given an arbitrary state of the SOE and the state of the monetary-union in period  $t$ ,  $\mathcal{S}_t$  and  $\mathcal{M}_t$ , we can express the following probabilities:<sup>20</sup>

$$prob_t(X_{t+1} = 1|\mathcal{S}_t; u, n) = prob_t(X_{t+1} = 1|\mathcal{S}_t; G(C)), \quad (2.27)$$

$$\begin{aligned} prob_t(X_{t+1} = 1|\mathcal{S}_t; u, s) &= p [\zeta prob_t(X_{t+1} = 1|\mathcal{S}_t; C^L) + (1 - \zeta) prob_t(X_{t+1} = 1|\mathcal{S}_t; C^H)] \\ &\quad + (1 - p) prob_t(X_{t+1} = 1|\mathcal{S}_t; C^e). \end{aligned} \quad (2.28)$$

Thanks to the additive nature of the exit-utility loss we can simplify equation 2.27 to the form

$$prob_t(X_{t+1} = 1|\mathcal{S}_t; u, n) = prob_t(X_{t+1} = 1|\mathcal{S}_t; C^e). \quad (2.29)$$

We can immediately observe that if

$$\zeta prob_t(X_{t+1} = 1|\mathcal{S}_t; C^L) + (1 - \zeta) prob_t(X_{t+1} = 1|\mathcal{S}_t; C^H) \neq prob_t(X_{t+1} = 1|\mathcal{S}_t; C^e),$$

then an “exit-rumors shock” at the union-level in period  $t$  would have an effect on the government's probability to exit the union in period  $t + 1$ . Importantly, as a relevant case for our analysis, given a particular  $\mathcal{S}_t$  if decision-makers would set their expectations such that

$$prob_t(X_{t+1} = 1|\mathcal{S}_t; C^e) = 0, \text{ and} \quad (2.30)$$

$$prob_t(X_{t+1} = 1|\mathcal{S}_t; C^L) > 0, \quad (2.31)$$

then (2.30) and (2.31) imply

$$prob_t(X_{t+1} = 1|\mathcal{S}_t; u, s) > prob_t(X_{t+1} = 1|\mathcal{S}_t; u, n),$$

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<sup>19</sup>Assuming  $\tilde{p}_s > 0$  would reinforce the qualitative results presented in this section further in expense of notational burden.

<sup>20</sup>We would like to note that  $prob_t(X_{t+1} = 1|\mathcal{S}_t; u, n) > 0$  is a theoretical possibility for the SOE, although - at the level of the monetary union in the absence of current exit-rumors - the exit of any other member state is not possible.

for any  $p, \zeta > 0$ , because  $p\zeta \text{prob}_t(X_{t+1} = 1|\mathcal{S}_t; C^L) > 0$  for all  $p, \zeta > 0$  and  $\mathcal{S}_t$  that we concentrate on. Hence, we obtain the following important qualitative property.

**Proposition 2.3.1** *If conditions (2.30) and (2.31) hold, then an exit-rumors shock in period  $t$  increases the likelihood of the SOE exiting the union in period  $t + 1$ .*

The intrinsic motivation for the government of the SOE to consider an exit from the union is associated with the benefits from setting the nominal exchange rate independently as such to devalue its own domestic currency relative to the currency of the monetary union. Specifically, in our framework, as in Schmitt-Grohé and Uribe (2016) and Na et al. (2018), the downward nominal rigidity in wages induces currency devaluation to be desirable during economic downturns in order to relieve the burden of unemployment and to reduce the output losses caused by a strong exchange-rate peg. The key feature distinguishing our model from these two studies is that in our set-up devaluation is not a costless action, because in our framework the country needs to exit the monetary union first to be able to devalue.<sup>21</sup> As an immediate implication of our model, we formalize the following key remark.

**Remark.** *Since the fundamental reason that motivates paying the exit cost  $\tilde{C}$  is to devalue the currency, in our framework departing from the union always comes along with a currency devaluation and a follow-up debt conversion.*

This property creates immediate implications of an exit-rumors shock for the cost of external borrowing. Specifically, let us first assume that the government cannot (fully) default on its external debt, such that  $D_t = 0$  for all  $t$ . However, partial default is still possible and upon exiting the union, the government is expected to execute it with certainty as highlighted in the remark above. The price of external debt in period  $t$  ( $q_t$ ) is determined by equation (2.24), which takes the probability of a devaluation between periods  $t$  and  $t + 1$  into account. Then, for those  $\mathcal{S}_t$  for which (2.30) and (2.31) hold, we have

$$q_t(\mathcal{S}_t; u, s) < q_t(\mathcal{S}_t; u, n). \quad (2.32)$$

The property (2.32) arises, because (i) in the period of a monetary-union exit the government devalues its currency, which affects the repayment of external borrowers, and (ii) if (2.30) and (2.31) are satisfied, the exit-rumors shock generates a likelihood of exiting the union over the near-horizon for the SOE. Therefore, at the onset of a “Grexit” type rumors - without necessarily the realization of an actual exit - the borrowing interest rates ( $1/q_t$ ) are expected to rise also for countries which did not necessarily experience a drastic deterioration in country-specific fundamentals. This property helps us to qualitatively capture the stylized

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<sup>21</sup>We would like to note that the downward wage rigidity is one micro-founded way of inducing devaluation to be optimal. There are other rationale why devaluation could be an optimal policy during an economic downturn. Our key mechanism does not require downward rigidities per se. What it requires is the presence of a motive to devalue the currency in certain states of the world, such that when the cost of the monetary union exit is low enough, the government would find abandoning the currency union and adopting a devalued domestic currency welfare improving.

fact of rising interest rates in Southern European countries following the Greek debt crisis, as depicted in Figure 2.1. We summarize this important property in the next proposition.

**Proposition 2.3.2** *If conditions (2.30) and (2.31) hold, then an exit-rumors shock in period  $t$  causes the interest rates to rise on newly issued SOE-bonds in period  $t$ .*

This qualitative channel is re-inforced, because the government is also allowed to fully default on its debt. The intuition is as follows: the rising interest rates (or falling bond prices) caused by the heightened likelihood of an exit-and-devaluation sequence increases the future repayment burden for the government of the SOE, which as in any other model of sovereign default leads to a further surge in interest rates.

An important condition that gives rise to the findings of propositions 3.1 and 3.2 and one that motivates our quantitative analysis in the next section is  $prob_t(X_{t+1} = 1 | \mathcal{S}_t; C^e) = 0$ . This turns out to be an empirically well-justifiable condition: before the Greek crisis hit the eurozone, the interest rates charged on sovereign bonds of Southern European countries equaled to the risk-free interest rates charged on sovereign bonds of Germany. This we interpret as that the external lenders did not forecast any short-run possibility of a euro-exit before the Greek sovereign debt trouble, which in the next section will help us in assigning a benchmark value for the expected cost of exit. Using our notation, this means that  $1/q_t(\mathcal{S}_t; u, n)$  equaled to the risk-free interest rate, implying  $prob_t(X_{t+1} = 1 | \mathcal{S}_t; C^e) = 0$ . Moreover, as an additional motivation in the midst of the crisis the unemployment rates in Southern European countries surged and these countries fell into deep and prolonged recessionary episodes. Even then, no member state decided to exit EMU in an attempt to regain international competitiveness and to reduce unemployment.

## 2.4 Quantitative Analysis

In this section we parameterize the model and solve it numerically in order to explore the dynamics of the small open economy, the SOE, around the times of an exit-rumors shock. In particular we are interested in studying the adjustments in default and exit likelihood of the SOE's government following an incidence of exit-rumors in the monetary union and the reaction of external lenders to such adjustments. For this purpose at first we calibrate a baseline economy where the likelihood of exit-rumors at the union level is zero. Specifically, in the baseline quantitative framework we impose a monetary union history such that (i) there were no exits from the union in the past, (ii) there are no rumors about a future exit of a member state and (iii) the probability of such news appearing in the near future equals zero. Analyzing the behavior of an economy without past-exits and exit-rumors allows us to study decision making solely based on the SOE's country-specific shocks on tradable output, past debt decisions and the level of nominal wages in the SOE.

After having analyzed the baseline dynamics of the model, we will turn to investigating the potential of sovereign debt contagion generated by a first-time exit-rumors shock stem-



ming from a troubled member state. For this follow-up analysis, we will conduct quantitative experiments using the calibrated framework, where we will surprise the monetary-union member (SOE) with the arrival of the news about a (first-time) potential departure of another member state in the near future and investigate the aggregate dynamics governing the SOE after this exit-rumors shock.

### 2.4.1 Functional Forms and Calibration

In Table 2.1 we provide the details of the benchmark parametrization of the model. Our parametrization largely resembles that of Na et al. (2018), where we follow their calibration for Argentina, since the quantitative analysis is meant to illustrate the dynamics of the model, rather than to replicate the actual data. Concentrating on the Argentinian calibration allows us also to consider a scenario of an Argentina-like country being a member of the EMU and going through the experience of the recent crisis. Furthermore, Argentina has a well-documented history of modern-times defaults, contrary to any of the actual EMU members, which makes it possible to calibrate the parameters related to sovereign debt and default. The details of the model parametrization are as follows.

We fix the labor endowment to unity and set  $\gamma = 0.99$ , which also Na et al. (2018) choose based on the evidence provided in Schmitt-Grohé and Uribe (2016), implying that nominal wages cannot fall more than 4 percent a year. To the end of functional forms, we assume a CRRA type of utility function as

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \quad (2.33)$$

and choose  $\sigma = 2$ . For the discount factor we assign a value of  $\beta = 0.87$ , which is somewhat lower than the standard parametrization of the discount factor in macroeconomic models, but not unusual for an Eaton-Gersovitz type of set-up. In the consumption aggregator we assign the share of tradables as  $a = 0.28$  and the elasticity of substitution between tradables and non-tradables as  $\varepsilon = 0.44$ . For the production technology of the non-traded sector, we specify

$$y_t^N = h_t^\alpha$$

and assume  $\alpha = 0.59$ . To the end of the output process, we use the Na et al. (2018) OLS estimates of (2.17) given by  $\rho = 0.932$  and  $\sigma_y = 0.037$ .

We specify the output loss function that is relevant in states of default as

$$L(y_t^T) = \max(0, \delta_1 y_t^T + \delta_2 (y_t^T)^2)$$

and set  $\delta_1 = -0.25$  and  $\delta_2 = 0.27$ . These parameters are chosen to achieve a default frequency

Table 2.1: Benchmark Parametrization of the model

Parameter	Value	Description
$\gamma$	0.99	Degree of downward nominal wage rigidity
$\sigma$	2	Inverse of intertemporal elasticity of consumption
$y^T$	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
$a$	0.28	Share of tradables
$\varepsilon$	0.44	Elasticity of substitution between tradables and non-tradables
$\alpha$	0.59	Labor share in the non-traded sector
$\beta$	0.87	Quarterly discount factor
$r^*$	0.01	Quarterly net world interest rate
$\delta_1$	-0.25	Parameters of the output loss functions
$\delta_2$	0.27	
$\rho$	0.932	Serial correlation of $\ln y_t^T$
$\sigma_y$	0.037	Standard deviation of innovation to $y_t^T$
$C^L$	0.8	Low exit cost
$C^H$	3.8	High exit cost
$C^e$	2.3	Expected cost of exit
$\tilde{p}_n$	0	Probability of a rumors shock
<b>Discretization of the state space</b>		
$n_y$	31	Number of tradable output grid points (equally spaced in logs)
$n_d$	101	Number of debt grid points (equally spaced)
$n_w$	151	Number of wage grid points (equally spaced in logs)
$[y^T, \bar{y}^T]$	[0.65, 1.53]	Grid for tradable output
$[d, \bar{d}]$	[-0.5, 1.25]	Grid for external debt
$[w, \bar{w}]$	[0.9, 5.15]	Grid for nominal wages

of 1.8 times a century under a flexible exchange rate regime.<sup>22</sup> We choose the risk-free rate as  $r^* = 0.01$  per quarter, a commonly assigned value in the literature.

Finally, we specify the expected utility cost of exiting the monetary union and the “high-cost” and “low-cost” realizations of this exit cost. The calibration of these parameters is challenging, as no country has ever exited the EMU. We use this observation to choose a benchmark expected cost of exit, such that exit does not happen in simulations in which the SOE takes the expected utility cost figure as given. We choose the low cost realization in such a way that exit could happen in *bad times*. This parametrization should reflect well the situation governing the EMU. We assume that there are two possible exit-cost realizations, denoted with  $C^H$  and  $C^L$ , both occurring with equal likelihood, and set  $C^H = 3.8$  and

<sup>22</sup>The default frequency implied by our framework is somewhat lower than the actual value for Argentina of 2.6, and the values for  $\delta_1$  and  $\delta_2$  differ from the values chosen by Na et al. (2018). The lower penalty values stem from the fact that there is no possibility to reenter financial markets after default, hence the output loss is permanent.

$$C^L = 0.8.$$

In order to provide an interpretation for the utility loss associated with the union-exit, we perform a welfare analysis. At first we specify a counterfactual regime, comparable to our AUTARKY regime in the sense that the economy cannot lend or borrow internationally, and that it has a flexible exchange rate in such a way that there are no nominal frictions. However, a key distinction from the AUTARKY is that there is no output loss. We then compute the welfare of this economy in a stationary equilibrium and quantify permanent contractions in consumption - and associated loss in utility from consumption - that would be in (net present value) equivalent to the utility loss from exiting the union.<sup>23</sup> The quantitative results reveal that our benchmark low-cost exit disutility scenario of  $C^L = 0.8$  is equivalent to a utility loss associated 9.4% permanent contraction in consumption, while the high-cost exit scenario of  $C^H = 3.8$  is equivalent to a 33% permanent contraction in consumption. These values may appear large, but they are not unrealistically high when compared to the tremendous output losses and unemployment spikes experienced in some of the eurozone countries during the recent crisis years. Moreover, these measured welfare losses need to be interpreted as gross (and not net) cost of the union-exit: an actual exit corrects the exchange rate misalignment, relieves the binding wage rigidity constraint and also lowers the real external debt burden through a partial default. All these feedbacks would generate income gains, which we do not take into account when quantifying the consumption-loss-equivalent of exit-costs.<sup>24</sup>

We solve the model by value function iteration and approximate the equilibrium using a discrete state space by assuming 31 grid points for tradable output, 101 grid points for debt and 151 grid points for nominal wage.

## 2.4.2 Equilibrium Dynamics without Exit Rumors

We first analyze the equilibrium dynamics of a baseline economy, where exit rumors are fully absent. To be precise, we specify that there were no previous exits, there are no exit rumors and the probability of exit rumors and any exit-realization emerging in the future equal zero. In terms of the model specification this is equivalent to the state of the monetary union of  $\mathcal{M}_t = (u, n)$ , i.e. no past exits ( $h_t = u$ ) in combination with no rumors about a potential exit ( $e_t = n$ ) and no exit-rumors possibility in the future ( $\tilde{p}_n = 0$ ).

Shutting down the shocks stemming from the state of the monetary union allows to focus on the dynamics of the SOE in isolation of monetary union dynamics and provides with a baseline to compare our framework with the existing literature on sovereign default. The properties we will establish in this section will also serve as a reference point for the full-fledged quantitative framework where the SOE will be shocked with monetary-union exit-rumors of another member state.

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<sup>23</sup>We measure the welfare by the value of the value function for the steady state level of tradable output, which is the only state variable for this problem.

<sup>24</sup>It is also important to highlight that exit cost does not need to be equal to the benefit of joining the union. Most likely, the loss of the benefits of being part of the union is only part of the exit cost, whereas important exit-cost items might be associated with the disruptive processes triggered by exiting the union.

We run 1000 simulations, each covering 5000 quarters. In every simulation, as initial conditions the SOE starts with zero net foreign assets, a low enough nominal wage - such that the nominal rigidity is not binding initially - and an average level of tradable output. Furthermore, the SOE is a member of the monetary-union and has not defaulted on its external debt in the past. This means that the SOE starts out in the UNION regime. In our numerical exercises that we report in Tables 2.2 and 2.3, we explore the likelihood of the SOE switching from the UNION regime into the regimes of EXIT (exiting the union, but honoring debt obligations), DEFAULT (defaulting on external debt, but remaining in the union) and AUTARKY (both defaulting and exiting) over the 5000-quarter time horizon in 1000 baseline simulations.

Table 2.2: Expected exit costs and the frequency of regime switches as a fraction (%) of 1000-simulation series

Expected exit cost	Consumption equiv. of exit util. cost	Fraction of simulations with regime switch		First regime switch		
		EXIT	DEFAULT	EXIT	DEFAULT	AUTARKY
0.7	8.3%	99.7	100	66.2	33.2	0.6
0.8	9.4%	93.9	100	53.5	46.1	0.4
1.1	12.5%	33.0	100	22.8	77.1	0.1
1.5	16.3%	3.9	100	3.5	96.5	0
1.9	19.8%	0.1	100	0	100	0
2.2	22.2%	0.0	100	0	100	0
<b>2.3</b>	<b>23.0%</b>	<b>0.0</b>	<b>100</b>	<b>0</b>	<b>100</b>	<b>0</b>

Table 2.2 reports the incidence of regime switches in our simulations. The first column presents the level of the expected cost of exiting the union, where we present the case of our benchmark  $C^e$  expected utility cost parametrization in the last row. The second column provides the permanent consumption-loss equivalent of the expected exit cost. As reported in column 3 of Table 2.2, on the one hand, over the horizon of 5000 quarters (1250 years) the SOE executes “a default” in 100% of the 1000-simulation series, independent of the expected cost of exiting the union. On the other hand, the fraction of simulation series, in which “an exit” occurs over the 1250-year time-horizon, crucially depends on the expected exit cost: an exit occurs in 99.7% of 1000 simulations under the lowest value of the expected exit-cost ( $C^e = 0.7$ ) while no exits can be observed for high enough values of expected cost of exiting, to be precise for any  $C^e \geq 2.2$ . These results reveal that the model exhibits a high degree of non-linearity, where relatively small changes in the expected exit-cost can lead to large changes in the likelihood of executing a monetary union exit.

Another interesting dimension to investigate is the sequence of regime switches, i.e. an analysis of whether a default or an exit occurs first in the SOE which starts out in the UNION regime. We explore this in the last three columns of Table 2.2. The last column of Table 2.2 reveals that it is very rare that exit and default happen simultaneously in the same time period, implying a low probability of switching directly to AUTARKY. The impact of the cost of exit on the ordering of switches is as expected: the lower the exit cost the more likely it becomes for the SOE to exit the union first before defaulting on its external debt. The non-linearity of the model is also very well visible in the sequence of regime switches: a moderate change in the exit cost may lead to a substantial change in the likelihood of observing an exit first before default.<sup>25</sup>

Within our model, whenever an exit happens it is followed by a devaluation. The average devaluation accompanying a switch from the UNION to the EXIT regime ranges from 50.4% (the case of  $C = 0.7$ ) to 62.5% (the case of  $C = 1.5$ ). The average devaluation increases with the exit cost. This is an endogenous feature of the model, as with a higher exit cost the welfare loss from the nominal friction has to be larger for the country to pursue an exit. Hence, with a higher exit cost there are less exits but they happen under worse economic conditions, implying also a larger devaluation. Given the re-denomination of sovereign debt by the time of an exit, the size of the devaluation also represents the losses suffered by external lenders.

Table 2.3: Expected exit costs and unconditional and conditional regime switch probabilities in baseline simulations (quarterly probabilities expressed in %).

Expected exit cost	Unconditional probability of exit (full sample)	Probability of exit after default (in the first)			Unconditional probability of default (full sample)	Probability of default after exit (in the first)		
		1y	2y	5y		1y	2y	5y
0.7	0.20	1.13	0.83	0.44	0.32	1.06	0.79	0.48
0.8	0.08	0.70	0.46	0.30	0.30	1.07	0.77	0.49
1.1	0.01	0.42	0.28	0.13	0.27	0.88	0.60	0.55
1.5	0.00	0.05	0.03	0.02	0.24	0.71	0.36	0.29
1.9	0.00	0.03	0.01	0.01	0.24	0.00	0.00	0.00
2.2	0	0	0	0	0.38	NA	NA	NA
<b>2.3</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.38</b>	<b>NA</b>	<b>NA</b>	<b>NA</b>

\*“0” refers to  $prob. = 0$ , “0.00” refers to  $0 < prob. < 0.01$ , and “NA” refers to conditional probabilities that could not have been calculated as the condition event did not occur in baseline simulations.

Importantly, using the baseline quantitative framework we can also study the complementarity between exit and default. The numerical analysis of Table 2.3 reveals that default

<sup>25</sup>Such as what we observe between the second and the third rows of Table 2.3.

increases the probability of a subsequent exit from the monetary union: for instance, when focusing on the lowest expected cost of exit, the probability of the monetary-union exit within a year after a default is more than five times higher than the unconditional probability of a monetary-union exit. This complementarity becomes stronger as the expected cost of union-exit increases (up until the point where no exits occur).

The mechanism behind this complementarity works through the output loss of default: after a default incident the SOE suffers an output loss and cannot smooth its consumption because of the exclusion from financial markets. Therefore, following default the optimal wage in the economy falls, increasing the likelihood of the nominal wage constraint to bind and lowering international competitiveness borne by the fixed exchange rate. Since the output loss is persistent, the nominal rigidity may generate persistent unemployment.<sup>26</sup>

The reverse causality - although likely - is quantitatively less strong: Depending on the expected cost of exit, a monetary-union exit increases the subsequent probability of default within a year by three to four times. The quantitative effect of an exit on default is smaller than that of default on exit because exiting the union already features a partial default, reducing the need for an outright default on external lenders.

### 2.4.3 Equilibrium Dynamics with Exit Rumors

In the previous subsection, we explored the baseline equilibrium dynamics in which exit-rumors are absent and given the unknown cost of union-exit the SOE never executes a monetary union departure as long as  $C^e$  is high enough. We are now ready to explore the consequences of exit-rumors. For this analysis at first we set  $C^H = 3.8$  and  $C^L = 0.8$  (with  $C^e = 2.3$ ) as a benchmark such that in the absence of exit rumors the country would never execute a monetary union exit by itself for any given  $\mathcal{S}_t$  that we concentrate on. In our quantitative analysis that we present in Table 2.4 we also study several alternative exit-cost specifications. As a benchmark we set a high enough value for  $C^e$ , because until the emergence of Grexit rumors no country had ever considered to leave the European Economic and Monetary Union. Moreover, exit-risks were never priced in the market for European sovereign bonds, as sovereign bond prices of peripheral EMU countries equaled to the risk-free rates charged on German bonds until 2009.

We surprise the SOE characterized by the baseline simulation dynamics (with  $C^e = 2.3$ ) with an exit-rumors in order to explore the dynamic implications of an unexpected exit-rumors shock. Specifically, we let the state of exit-rumors in the monetary union ( $e_t$ ) unexpectedly change from  $n$  to  $s$ . This means that after the rumors shock is realized, there will be another member state potentially exiting the union before the next period begins. From this point on we will continue to refer to the country whose macroeconomic behavior we are studying as the SOE, and the member-state that is the subject of exit rumors as country-X.<sup>27</sup>

<sup>26</sup>This resembles the optimality of devaluation following the incidence of default in Na et al. (2018).

<sup>27</sup>In the case of the European crisis, one can think of Greece as country-X and any of the peripheral countries affected by contagion, e.g. Portugal, as the SOE.

For tractability purposes, we assume that the “perceived” *ex-ante* probability that country-X actually exits the union ( $p$ ) equals 1, whereas ex-post an exit does not materialize. This property requires some level of myopia for the agents of the model. This assumption substantially simplifies computations, but is of no qualitative importance.<sup>28</sup>

Table 2.4: Default Implications of Exit Rumors

Experiment setup			Exit Prob. (%)	Additional defaults generated by Exit-Rumors as % of <u>no-default periods</u> in baseline sim.		
$C^e$	$C^L$	$C^H$		all periods	periods with $d_t > 0$	periods with $d_t > 0, w_{t-1} > \bar{w}$ & $y_t^T < y^T = 1$
<b>2.3</b>	<b>0.8</b>	<b>3.8</b>	<b>0</b>	<b>0.78</b>	<b>1.44</b>	<b>4.43</b>
2.3	0.7	3.9	0	1.20	2.21	6.20
2.3	1.1	3.5	0	0.42	0.78	1.92
2.3	1.9	2.7	0	0.25	0.46	0.38
1.5	0.7	2.3	0.00	0.75	1.38	5.06
1.5	0.8	2.2	0.00	0.43	0.79	4.08
1.5	1.1	1.9	0.00	0.16	0.30	1.52

\*  $\bar{w}$  is the median equilibrium nominal wage that reveals in baseline simulations.

The arrival of exit-rumors stemming from country-X does not have a direct consequence for the SOE, as it does not change its aggregate state  $\mathcal{S}_t$ . However, rumors shock has substantial indirect effects through the possibility of exit-cost information revelation before the next period actions are taken, which will have dynamic implications for the SOE and in particular for its current bond prices.

How exactly does the mechanism of the model work? Without exit rumors, it is impossible for the exit cost to be revealed (exogenously) in the next period; therefore, both the SOE and international investors know that the SOE’s government will undertake its decisions based on the expected exit cost,  $C^e = 2.3$ . This implies that the exit likelihood of the SOE equals zero and the exit-driven partial-default risk does not need to be priced in when extending funds to the SOE’s government in international financial markets.

If there is a positive chance that the cost of exit is going to be revealed before the next-period decisions are taken by the SOE, the pricing of bonds gets revised. The reason for this adjustment - as we delineated before - is that in that case the next-period decisions are going to be determined based on one of the two possible cost realizations, either with  $C^L = 0.8$

<sup>28</sup>Quantitatively the myopia assumption increases the contagion effect. However, given the large contagion effects that we find through the rumors shock, it might be the case that modeling the rumors shock without myopia could bring our results closer to reality.

or with  $C^H = 3.8$ . If the realized cost figure will turn out to be  $C^H$ , the SOE's monetary union exit likelihood will not exhibit any change from that of the baseline. The reason for this invariance is that for any  $\mathcal{S}_t$ , the SOE was not going to execute a union-exit given the expected utility cost of  $C^e = 2.3$ , as we analyzed in the baseline equilibrium dynamics. Hence, by taking a realized cost figure of  $C^H$  that is higher than  $C^e = 2.3$  as given, it will not find a monetary-union departure optimal either. However, if the realized cost figure will be  $C^L$  (with  $C^L < C^e = 2.3$ ), the SOE might find it optimal to leave the union for some  $\mathcal{S}_t$  - as long as  $C^L$  is low enough.

Table 2.5: Default Multipliers

Experiment setup			Default Probability with Exit Rumors Default Probability without Exit Rumors		
$C^e$	$C^L$	$C^H$	all periods	periods with $d_t > 0$	periods with $d_t > 0, w_{t-1} > \bar{w}$ & $y_t^T < y^T = 1$
<b>2.3</b>	<b>0.8</b>	<b>3.8</b>	$\frac{0.78+0.38}{0.38} = 305\%$	$\frac{1.44+0.70}{0.70} = 305\%$	$\frac{4.43+1.00}{1.00} = 543\%$
2.3	0.7	3.9	$\frac{1.20+0.38}{0.38} = 415\%$	$\frac{2.21+0.70}{0.70} = 415\%$	$\frac{6.20+1.00}{1.00} = 720\%$
2.3	1.1	3.5	$\frac{0.42+0.38}{0.38} = 210\%$	$\frac{0.78+0.70}{0.70} = 211\%$	$\frac{1.92+1.00}{1.00} = 292\%$
2.3	1.9	2.7	$\frac{0.25+0.38}{0.38} = 166\%$	$\frac{0.46+0.70}{0.70} = 166\%$	$\frac{0.38+1.00}{1.00} = 138\%$

\*  $\bar{w}$  is the median equilibrium nominal wage that reveals in baseline simulations. The denominators in fractions are the default probabilities without exit rumors.

Since the primary reason to incur an exit utility-loss is to correct the exchange rate misalignment of the country, a future exit from the monetary union will come along with the introduction of a new currency and an immediate re-denomination of the stock of outstanding debt into this new currency, making a devaluation equivalent to a partial default. Therefore, when exit-rumors are present, expected losses on international lenders that would materialize in the next period after a potential  $C^L$  revelation are taken into account when pricing the SOE-bonds in the current period.

Our mechanism thus yields that even in the absence of a shock to the aggregate state of the SOE ( $\mathcal{S}_t$ ), the bond prices of the SOE should go down. Low bond prices, or in other words high interest rates, in turn could induce an outright default to be optimal for the SOE's government, even if the government does not choose to execute a monetary-union exit in any time-period soon before or soon after the default. We interpret defaults triggered by exit-rumors shocks - in the absence of additional shocks to fundamentals and any realized exit from the union - as contagion within the monetary union. Since the fall in bond prices is the result of a potential future union-exit under a low exit-cost realization, the contraction



in bond prices gets magnified by the benefits of a potential exit. Therefore, the contagion is more likely to occur and to be more pronounced in countries that have high debt levels and/or experience a binding nominal wage rigidity constraint. Next we turn to analyzing these channels quantitatively.

First, Tables 2.4 and 2.5 analyze the implications of exit-rumors on the SOE’s default likelihood, in which we delineate on the number of new default incidences that will emerge after the exit-rumors shock. Specifically, in Table 2.4 we present the fraction of periods (as a percentage) of the baseline simulation periods, where the exit-rumors shock pushes the SOE to default, although no default was going to be observed in the absence of exit-rumors. As we illustrate in the top row of Table 2.4, given our benchmark parametrization with  $C^H = 3.8$  and  $C^L = 0.8$  (and  $C^e = 2.3$ ), the exit-rumors shock generates new sovereign default incidences in 0.78% of all simulation periods where default wasn’t going to be observed without the exit-rumors. Since default occurred (unconditionally) in 0.38% of all periods (quarters) in the baseline<sup>29</sup>, which continue to occur with exit rumors, as we report in Table 2.5, the unconditional probability of sovereign default with exit-rumors is 305% that of the baseline simulations without exit-rumors.

Furthermore, the exit-rumors shock increases the likelihood of default substantially more if the SOE is indebted, has lower than average output realizations and a binding wage constraint. Specifically, following an exit-rumors shock, new default incidences get reported in 1.44% of baseline simulation periods with positive debt. In periods when the SOE is indebted and also exhibits high wages coupled with lower than average tradable output - inducing the wage constraint to bind - we record new default incidences in 4.43% of the relevant range of periods. As we present in Table 2.5, these new default incidences imply that the probability of default with exit-rumors is 543% that of the baseline without exit-rumors, when one focuses on times of indebtedness coupled with a recession. Therefore, the exit-rumors mechanism in hand generates quantitatively important contagion effects for healthy members of a monetary union and disproportionately more so for those that are indebted and going through a recession, such as some of the countries in peripheral EMU right before the emergence of Grexit rumors.

Quantitative experiments in the second row of Table 2.4 indicate that keeping the expected exit-cost at  $C^e = 2.3$  and varying the degree of the spread between  $C^H$  and  $C^L$  (exit-cost uncertainty) does not alter the qualitative properties obtained in the top row of Table 2.4. Quantitatively the default implications of exit-rumors get substantially stronger if we were to increase the difference ( $C^H - C^L$ ) from 3.0 to 3.2, as we document in Table 2.5. Similarly, as we present in the third row of Table 2.4, lowering the expected-cost of exit from 2.3 to 1.5 does not alter the key qualitative properties either, while retaining strong quantitative implications.

Although with exit-rumors the SOE defaults more often than the baseline of “no-exit-rumors”, as indicated in Table 2.4 the government continues to remain in the monetary-union also with exit-rumors. This resembles the situation of peripheral EMU countries during the

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<sup>29</sup>The quarterly unconditional default probability of 0.38% is equivalent to an unconditional annual probability of 1.51%.

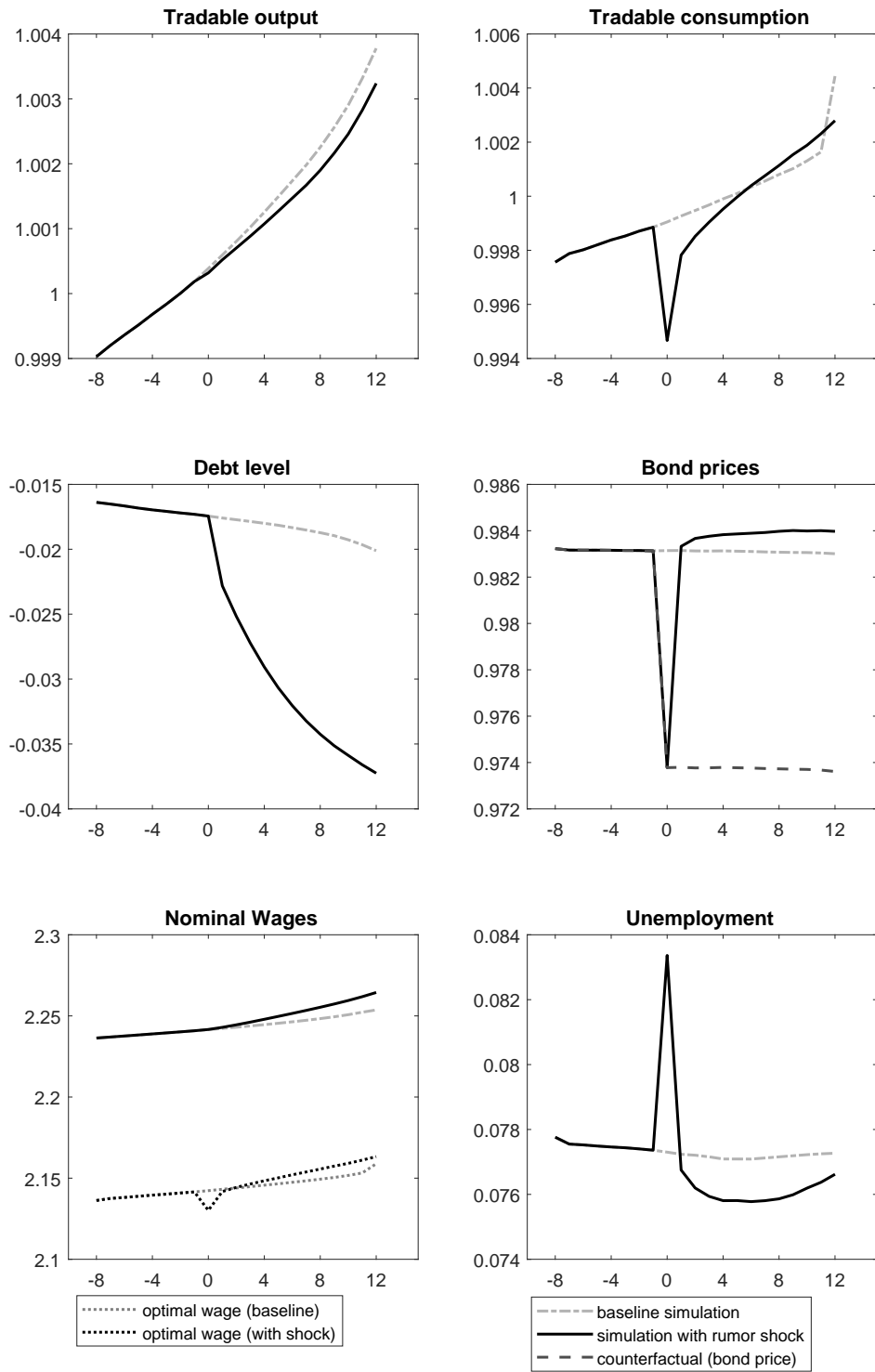


Figure 2.3: Simulation results - baseline simulation vs simulation with a rumors shock.

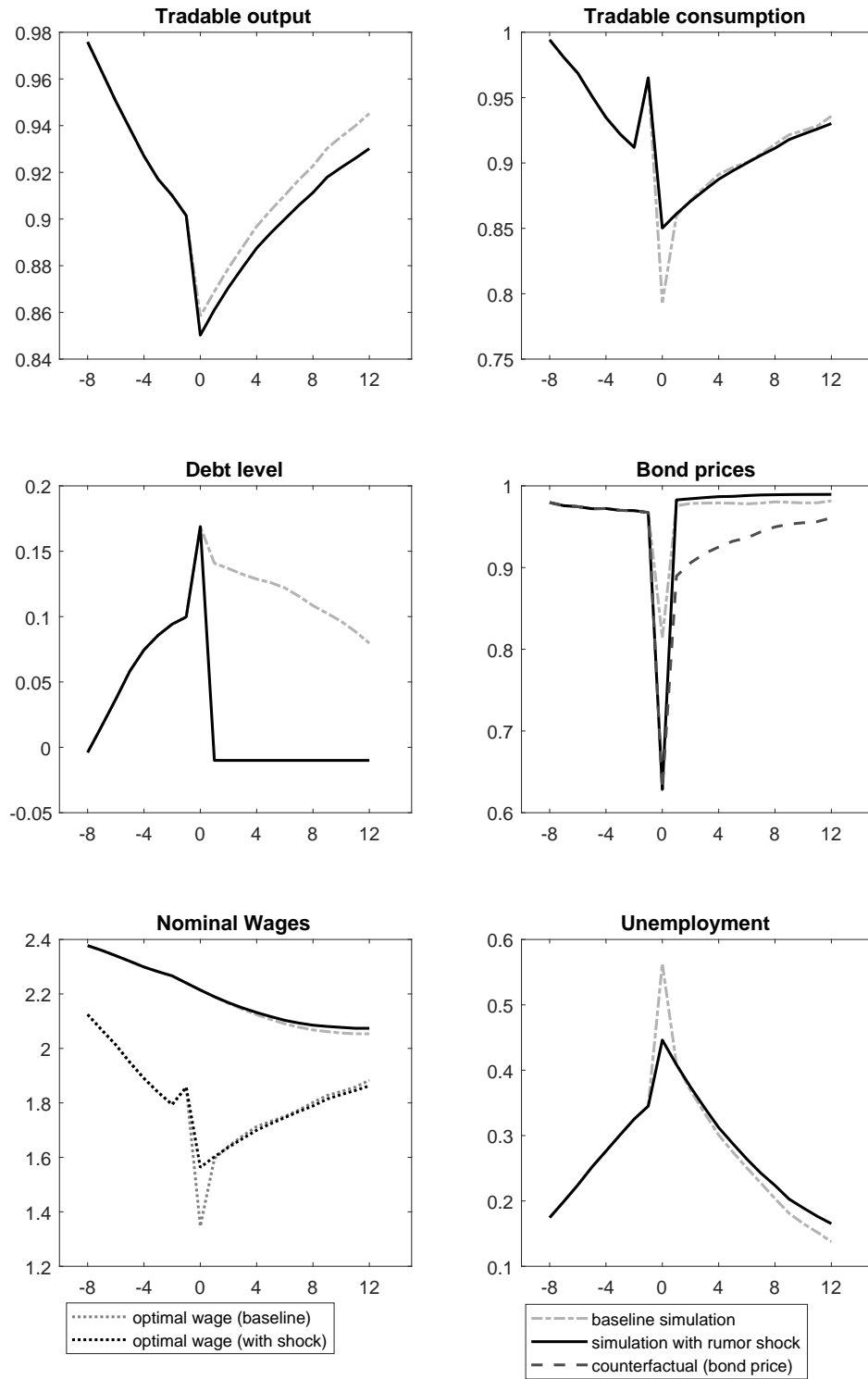


Figure 2.4: Simulation series in which there is no default in the baseline, but default happens after the rumors shock.

eurozone debt crisis: following the Grexit rumors, countries such as Portugal and Cyprus experienced contractions in prices on sovereign bonds and soon after they applied for bail-out funds. During this period neither of these countries attempted to exit the EMU.

Figures 2.3 and 2.4 present the behavior of a set of macroeconomic variables - averaged across simulations - for the benchmark exit-cost specification with  $C^H = 3.8$  and  $C^L = 0.8$ . Throughout the panels of both figures, the dashed-dotted gray lines represent the “average equilibrium values” across the 1000-baseline simulations without exit-rumors. In comparison, the solid black lines display the averages over simulation series, in which an exit-rumors hits the SOE in period zero.

The upper left panel of Figure 2.3 shows the dynamics of the tradable output, which exhibits a rising trend over the period that we concentrate on. However, looking at the scale reveals that the tradable endowment is virtually constant, as the change of its value over the period of 5 years is approximately one-tenth of the standard deviation.<sup>30</sup>

The effect of the exit-rumors shock becomes the most visible in the middle-right panel of Figure 2.3, where we study the dynamics of the SOE-bond prices: following the exit-rumors shock, the average price of SOE-bonds across all simulations falls. This reflects the perceived risk that an actual exit could happen in the near future for the SOE after exit-rumors, as a result of which its external debt could get re-denominated. The low bond prices, in turn, force the SOE to lower its debt level as it can be observed in the middle-left panel of Figure 2.3. The contraction in SOE’s indebtedness causes the consumption of tradable output to contract around the time of the exit rumors, which is depicted in the upper-right panel. The feedback through the adjustment in the level of indebtedness allows the SOE-bond prices to rebound a period after the exit-rumors shock hits the economy, even though the shock persists further. Finally, the dashed black line in the middle-right panel of Figure 2.3 displays the counterfactual bond price that would have prevailed after exit-rumors, if the SOE’s government were not to adjust the country’s external debt. In that case, as expected, the price on sovereign bonds remains low for a longer time-period, as it was experienced in peripheral EMU countries during the eurozone crisis.

A key feature of our model is the rigidity in nominal wages that would be dictated by a binding wage constraint. We can note in the lower left panel of Figure 3 that optimal wages in both baseline simulations (without rumors shocks) and in simulations with exit-rumors are lower than the realized wages in equilibrium, implying that the nominal wage constraint is binding in simulation-periods we are exhibiting. The contraction in tradable consumption following the exit-rumors impacts the non-tradable sector as well through its effect on nominal wages: the contraction in tradable consumption suppresses the price of non-tradables and as a result the optimal nominal wage level. Lower optimal wage rate induces the wage rigidity constraint to bind even more - translating into higher unemployment rates in the non-tradable sector as depicted in the lower-right panel of Figure 2.3.

In 0.78% of the simulation series the rumors shock triggers a default, where there would

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<sup>30</sup>The slightly upward trend in tradable output might result from the way we choose the sample, as we drop thirteen periods prior to the default-event in the baseline simulation and defaults happen mostly when output falls. If anything, this bias will underestimate the contagion effect that we are capturing. More details on sampling for the simulations is provided in Appendix A.

have been no default if exit rumors were not present, as we covered in the baseline simulations of Section 4.2. In these simulations default happens in the first period the rumors shock hits the economy. We interpret the additional default incidences generated by the exit-rumors as contagion. To delineate more on this contagion effect, Figure 2.4 presents the behavior of the macroeconomic variables averaged across those simulations in which there is a default in the first period the rumors shock hits, although no default is observed in the baseline simulations without exit-rumors. In this respect, this analysis concentrates on the simulations of the SOE, in which the SOE was under stress but healthy enough not to default in the absence of exit rumors. The SOE gets pushed to default after the arrival of exit-rumors. Therefore, in these special-case simulations the SOE's tradable output features a declining trend while its debt is rising before the exit-rumors hits, as presented in the upper-left and middle-left panels of Figure 2.4.

The simulation results that we observe in Figure 2.4 are qualitatively consistent with our findings in Table 2.4 such that the rumors shock is more likely to trigger a default if the economy is highly indebted, experiencing a recession and/or unemployment is high due to a binding nominal wage rigidity constraint.

Figure 2.4 shows that bond prices fall substantially after the exit-rumors hits the economy. We would like to note though that the bond prices would have been contracting for these sub-sample simulations even without rumors shock - as observed in the middle-right panel of Figure 2.4, and default possibility would have been priced, although no default was going to be realized ex-post. Due to the relatively high indebtedness of the SOE in these sub-sample simulations (with or without exit-rumors), the government does not find it optimal to reduce the country's debt level and instead it increases indebtedness further before the arrival of exit-rumors, causing a spike in the tradable consumption. When exit-rumors arrive and contract the bond prices further, it pushes the government of the SOE to default on its external debt. This channel increases the consumption of tradable output in the short run - relative to the baseline simulations without exit-rumors, because with the help of default the government avoids debt repayment to international investors. This standard property of Eaton-Gersovitz style models is qualitatively different than what we observed in the full-sample simulations of Figure 2.3. The elevation in tradable-consumption in the short-run is the main motive for the SOE's government to default, since defaulting allows the government to withhold its debt repayment. The relatively small contraction in tradable-consumption - compared to the full-sample simulations - also implies that the optimal price of non-tradables and the optimal wage rate fall less compared to the baseline, reducing short-term unemployment consequences of exit-rumors.

The costs of an outright default for the SOE are twofold: the output loss penalty and the exclusion from international financial markets. These two cost items reduce the consumption of tradable-output and keep it at those low levels in the periods following the incidence of default. This is also why the defaulting SOE experiences high levels of unemployment for several periods compared to the full sample simulations.

### 2.4.4 Uncertain Exit Costs and Cost of Borrowing

Exit-rumors shock deteriorates macroeconomic conditions of the SOE and potentially pushes the country's government to default to the extent it raises the cost of borrowing. In order to deepen our understanding concerning the effects of exit rumors on sovereign borrowing costs, in this section we vary the degree of uncertainty governing the union-exit cost and analyze the consequences of this variation on deviations in interest rates from the baseline interest rates by the time the exit-rumors shock hits the economy. We measure the potential “contagion effect” of exit-rumors by comparing the interest rate hikes caused by the exit-rumors - averaged across simulations - for different degrees of exit-cost uncertainty.

Figure 2.5 presents this analysis for the benchmark expected cost of exit,  $C^e = 2.3$ , by varying the degree of uncertainty across a spectrum of cost specifications and recording the deviations in average interest rates (from their baseline averages) following the exit-rumors shock. In our analysis we explore across a wide range of uncertainty specifications, starting from A1, with  $C^L = 0.7$  and  $C^H = 3.9$ , up to A7, with  $C^L = 2.2$  and  $C^H = 2.4$ .

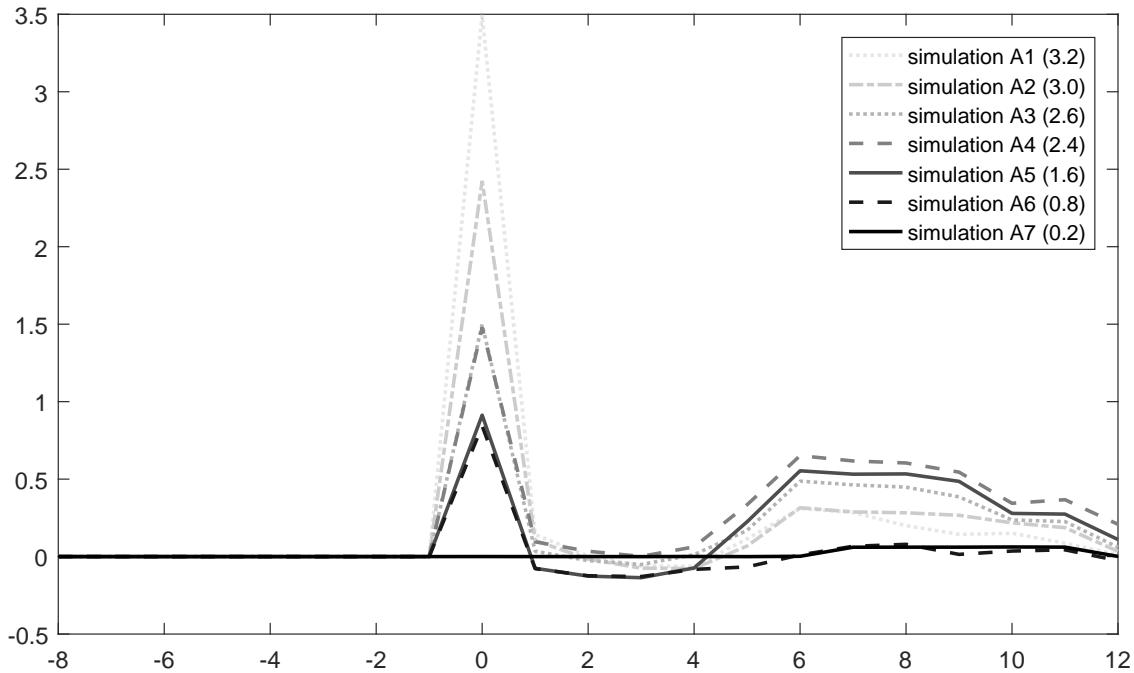


Figure 2.5: Contagion effect (rumors shock) on interest rates (in percentage points) for different exit cost parametrizations.  $C^e = 2.3$  is constant across different parametrizations,  $C^H - C^L$  reported in parentheses.

We would like to note that the A7-specification is an extreme case (where uncertainty is practically absent) which does not yield much of a fluctuation in interest rates during times of exit-rumors. This quantitative property holds, because for both  $C^L = 2.2$  and  $C^H = 2.4$  the SOE does not exit the monetary-union for any aggregate state of the country. For the remaining uncertainty specifications, at the onset of the exit-rumors quantitatively visible

deviations in interest rates get recorded. Furthermore, these short-run effects are larger the larger is the difference between  $C^H$  and  $C^L$ . The medium-run effect of exit-rumors on interest rates is not monotonic in exit-cost uncertainty, because the direct effect of the rumors-shock brings along an adjustment in external borrowing, reducing the default and exit risks.

We repeat the same analysis also for a range of uncertainty-specifications for  $C^e = 1.5$  and present the quantitative results in Figure 2.6. Similar qualitative and quantitative effects of exit-cost uncertainty on interest rates are observed for this alternative expected cost specification as well.

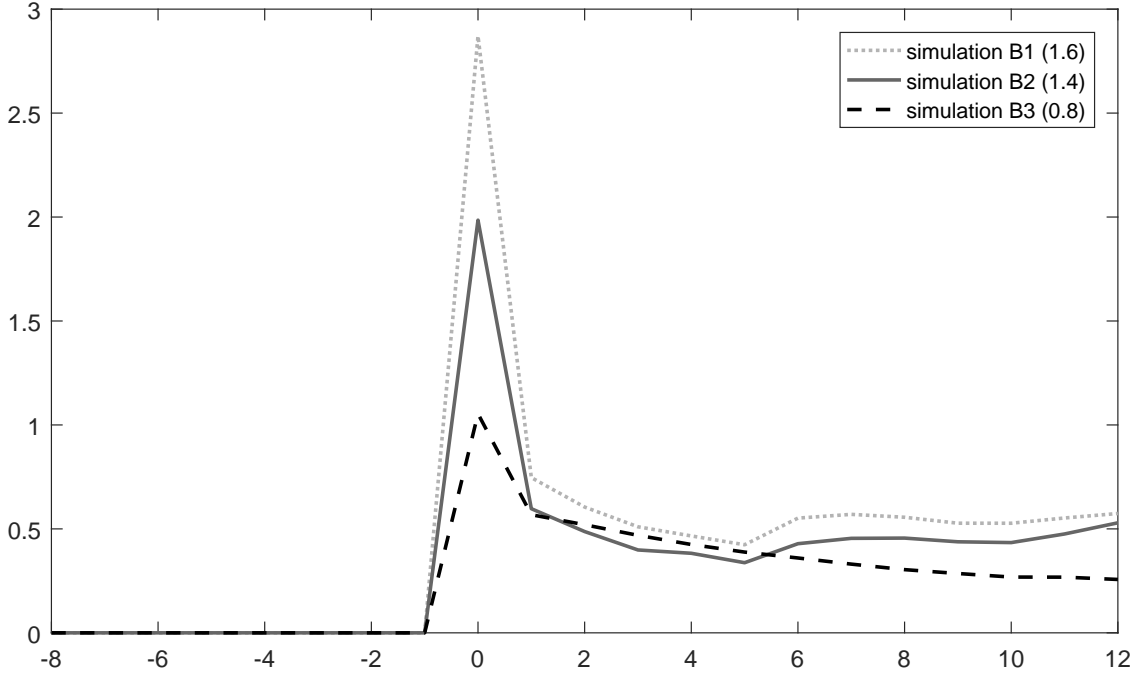


Figure 2.6: Contagion effect (rumors shock) on interest rates (in percentage points) for different exit cost parametrizations.  $C^e = 1.5$  is constant across different parametrizations,  $C^H - C^L$  reported in parentheses.

The results from this section have clear policy implications, as it shows that exit-cost uncertainty increases the fragility of the economies in a monetary union, implying that policy makers should aim for exit-cost transparency in order to eliminate financial instability and contagion in a currency-area.

## 2.5 Conclusion

We studied the interactions between default and monetary-union exit decisions of a small open economy. Our small open economy model features nominal wage rigidities, which induce exchange-rate misalignments to be painful for the macroeconomy as in Na et al. (2018)

and Schmitt-Grohé and Uribe (2016). Within this setup we analyze an Eaton and Gersovitz (1981) type endogenous default of the government on the country's external debt and importantly we endow the government of the small open economy also with an endogenous monetary-union exit decision. As key aspects of the set-up we specify that the union-exit is costly, and moreover the cost of exit is uncertain and becomes known to the members of the monetary-union only when a member state executes an actual departure from the union.

Using this model we uncover a novel contagion mechanism that could trigger a sovereign debt crisis (and default) in a country with healthy enough ex-ante fundamentals after the emergence of union-exit rumors associated with another member state despite the absence of any realization of a monetary-union exit. The mechanism in hand implies that the potential of a low exit-cost realization in the near future - such as a successful and not-so-painful Grexit process - is likely to generate a domino-effect and cause subsequent exits. Since a union-exit of the country of our interest in the future will also imply a partial default, external lenders take such short-run expectations into account around the times of an exit-rumors and raise interest rates, even for a not-so-unhealthy member state.

We calibrate the theoretical framework and present quantitative exercises that illustrate that the mechanism can replicate the qualitative properties of the sovereign debt crisis observed in EMU over the last decade. Moreover, the model accounts for rising interest rates in peripheral EMU countries - after the Grexit rumors - as well as high unemployment rates and economic slowdown in these countries during the eurozone debt crisis. Most importantly, we quantify that an exit-rumors shock would triple an average country's default likelihood, while it raises the default likelihood for a country which was already in a recessionary-trend to five times of its baseline-level. Therefore, our framework provides a novel and quantitatively important angle to evaluate the eurozone debt-crisis.

The contagion mechanism we describe is not the only possible source of contagion within the EMU. Other potential contagion channels include common lenders, wake-up call contagion resulting from the revision of Greek fiscal data in 2009-10, or the correlation of shocks across the union. What distinguishes the contagion channel that we present in this paper from the rest is that it corresponds well with the events observed during the eurozone crisis. In particular, contagion was limited only to the countries of the monetary union and the correlation of risk premia that emerged at the onset of the crisis persisted over time, with interest rates of the affected countries reacting to news about other eurozone countries. The former characteristic weakens the case for common lenders as the sole channel of contagion, whereas the latter characteristic makes the case for wake-up call contagion less likely.

Bail-outs could be another promising channel that could explain contagion within a monetary union. We perceive bailouts in a monetary union to be complementary to the channel presented in this paper, which potentially could reinforce the effects that we uncovered.

Our framework can be extended in a number of interesting dimensions. Among them, one policy relevant case would be the study of interactions between exit rumors emerging from different countries. In the theoretical and the quantitative analysis of the current paper we concentrated on the implications of exit rumors stemming from one member state to the macroeconomic dynamics and financial instability of another one. It could be interesting to



analyze the feedback loops between exit-rumors of different countries, which can reinforce the likelihood of short-run exit-cost revelation and financial contagion in the monetary union. Another interesting dimension to consider could be the incorporation of a central policy maker at the union-level and the analysis of its optimal reactions to contagion resulting from exit rumors. These interesting extensions go beyond the purpose of the current paper and therefore we leave them to future research.

## 2.6 Appendix A - Sampling for the simulations with an exit-rumors shock

This appendix describes the algorithm for creating the simulation sample for Section 4.3. In order to obtain the sample for the exit-rumors shock analysis, we utilize a subset of the baseline simulation series in which the monetary union channel, through shocks to  $\mathcal{M}_t$ , is totally shut down. The baseline that we presented in Section 4.2 consists of 1000 simulations, each lasting for 5000 periods. In order to obtain the exit-rumors simulation series of Section 4.3 we implement the following protocol and extract the necessary sample from the baseline simulation series.

1. We drop the first 30 periods (burn-in period) from each simulation series to avoid the influence of the initial conditions - characterized by average tradable output, no debt and low enough nominal wages.
2. We end each simulation series by the time-period when the first switch occurs, irrespective of whether the regime change is towards DEFAULT, EXIT or AUTARKY. This way we make sure that all the simulation points are in the UNION regime.
3. We cut the last twelve periods from each of the trimmed simulation series. This step is necessary to make sure that in twelve periods, following the period of an exit-rumors shock, there are no regime switches in the baseline simulation. This allows us to study not only the immediate effect of the rumors shock, but also its dynamic effects in subsequent periods.
4. We utilize all the remaining time-periods of the baseline simulation series.

In figures 3 and 4 we present the simulation results not only for the period, in which the economy gets hit by the exit-rumors shock, but also the eight preceding and twelve subsequent time-periods. These simulations are obtained in the following way:

- The preceding periods in the “exit-rumors simulations series” are simply copied from the baseline simulations, because prior to the shock nothing is different in the “exit-rumors series” from the baseline simulations, as the agents do not anticipate the exit-rumors shock.
- The period denoted with zero and the twelve subsequent periods in “exit-rumors series” follow the adjusted dynamics after the rumors shock, i.e. the agents in the economy take into account the possibility that before next period actions are taken the union-exit cost might get revealed.
- In order to isolate the effect of the rumors shock, in the “exit-rumors series” the exogenous process (i.e. tradable output process) over the whole time-horizon of each simulation series is imposed to take the same values as in the baseline simulations.

This way any difference between the simulation series with the rumors shock and the baseline simulation series will be solely driven through the adjusted behavior induced by the exit-rumors shock.

## 2.7 Appendix B - Definition of Recursive Equilibrium

The model's recursive equilibrium is defined by (i) decision rules  $D_t(\mathcal{S}_t, \mathcal{M}_t)$  and  $X_t(\mathcal{S}_t, \mathcal{M}_t)$  for the sovereign government with associated value functions  $V^U(\mathcal{S}_t, \mathcal{M}_t)$ ,  $V^X(\mathcal{S}_t, \mathcal{M}_t)$ ,  $V^D(\mathcal{S}_t, \mathcal{M}_t)$  and  $V^A(\mathcal{S}_t, \mathcal{M}_t)$ , consumption and transfer rules  $c_t(\mathcal{S}_t, \mathcal{M}_t)$ ,  $T_t(\mathcal{S}_t, \mathcal{M}_t)$ , debt policy  $d_t(\mathcal{S}_t, \mathcal{M}_t)$  and an exchange rate policy  $\tilde{e}(\mathcal{S}_t, \mathcal{M}_t)$ , (ii) a bond pricing function  $q_t(\mathcal{S}_t, \mathcal{M}_t)$ , and (iii) domestic pricing functions  $\mathcal{P}_t(\mathcal{S}_t, \mathcal{M}_t)$ ,  $w_t(\mathcal{S}_t, \mathcal{M}_t)$ , and a labor allocation  $l_t(\mathcal{S}_t, \mathcal{M}_t)$  such that:

1. Given (ii) and (iii), the decision rules  $X_t(\cdot)$  and  $D_t(\cdot)$  solve the government's maximization problems (20), (21), (22) and (23),
2. The consumption and transfer rules,  $c_t(\cdot)$  and  $T_t(\cdot)$ , satisfy the resource constraints of the economy, (3), (6) and (15),
3. Given government's decision rules, the bond pricing function,  $q_t(\cdot)$ , satisfies the no-arbitrage condition (24) for a country in the UNION-regime or the no-arbitrage condition (25) for a country in the EXIT-regime,
4. Given (i) and (ii), the private sector's pricing functions and labor allocation satisfy the competitive partial equilibrium (defined in section 3.2.2).

## 2.8 Appendix C - Timing of Events

The timing of events is presented in timelines 1 and 2. While Timeline 1 concerns the sequence of events in any time period  $t$  until which no exit from the monetary union had ever been observed, Timeline 2 depicts the timing in a period before which at least one of the member states decided to exit the union. In order to understand the dynamic implications of exit intentions of other member states on the SOE one needs both timelines 1 and 2. Timeline 1 can be conveniently divided into three stages, which follow one another. Events within each stage occur simultaneously:

- I. In Stage-I ex-ante shocks get realized.
  - The first shock is the realization of the level of tradables that is available for consumption and international financial market transactions of the SOE in period  $t$  ( $y_t^T$ ).
  - The second shock is related to the news about the monetary union. At this stage, news arrive whether at least one of the member states is considering to exit the monetary union. We interpret the Greek bail-out process in the eurozone and the Grexit rumors as the arrival of such news. We would like to emphasize that this news shock does not need to lead to the materialization of an actual exit - similar to the experience in the eurozone following the Greek debt crisis.
- II. In Stage-II, decision making of the key economic actors take place. Given the shocks realized in Stage-I:
  - The government of the SOE decides whether to stay in the union or not. If the decision is “to leave” the union, the country adopts its own domestic currency effective as of period  $t$ . The nominal exchange rate of the newly introduced currency gets determined by the government. If the currency is devalued at the on-set of this union-exit, this also implies a partial default on the existing debt.
  - Firms and households interact to produce the non-tradable output.
  - If the SOE was in good standing in period  $t - 1$  and carried over debt to be repaid in period  $t$ , the government can take a default decision on its outstanding debt. If the government defaults, the country immediately switches to a bad financial standing and its access to international financial markets gets cut.
  - If the SOE is in bad financial standing, it suffers  $L(y_t^T)$ .
  - If the SOE has good financial standing in period  $t$ , the government issues bonds to be repaid in period  $t + 1$ . The bonds are bought by the external lenders. The unit price of bonds is determined based on the no-arbitrage condition.
  - Households consume.
- III. If rumors about another member state’s exit intentions emerged in Stage-I, in Stage-III the news arrive concerning the materialization of an exit. There are three different alternative news to this end.

- If no country was seeking to exit in Stage-I, no exit is possible as of Stage-III (other than the possibility of an exit allowed for the SOE that we analyze in Stage-II).
- If another member state was seeking to exit in Stage-I, then this member state might exit in Stage-III and the cost of exit gets revealed immediately.
- Or, alternatively the exit decision of that particular member state, that was seeking exit opportunities in Stage-I, can get carried over to the next period. In this case that member state's exit intentions will be revised in Stage-I of period  $t + 1$ .

The timing of events that we present in Timeline 2 is close to Timeline 1, with the exception that at the beginning of Timeline 2 - since a union-exit had occurred at some point in the past - the realized cost of exit is taken as given by the SOE.

**Timeline 1: Sequence of Events in  $t$  - No Member State Exited the Union until  $t$**

- I - Ex-ante Shocks**
  - Tradables shock ( $y_t^T$ ) for the SOE.
  - The potential emergence of a country-X considering to exit the union.
- II - Decisions**
  - Government's union-exit decision by taking  $C^e$  as given.
  - If government exits, households incur the utility loss  $\tilde{C}$  only in  $t$ .
  - If it exits, the government decides the nominal exchange rate.
  - Firms hire labor supplied by households and produce.
  - If in good standing in  $t - 1$ , government's decision to default on debt from  $t$ .
  - If default, the country switches to bad standing.
  - Conditional on good-standing, foreign lenders price country's bonds.
  - Conditional on good-standing, government chooses debt to be paid in  $t + 1$ .
  - If in bad standing, loss of  $L(y_t^T)$ .
  - Consumption of tradables and non-tradables.
- III - Ex-post Shocks**
  - Exit decision of Country-X, if there was one considering to exit at Stage-I.
  - If country-X exits, realization of the value of  $\tilde{C}$ .
  - If country-X doesn't exit, it leaves its decision to  $t + 1$ .

**Timeline 2: Sequence of Events in  $t$  - One of the Member States Exited the Union before  $t$**

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**I - Ex-ante Shocks**

- Tradables shock ( $y_t^T$ ) for the SOE.

**II - Decisions**

- Government's union-exit decision by taking realized  $\tilde{C}$  as given.
- If government exits, households incur the utility loss  $\tilde{C}$  only in  $t$ .
- If exit, the government decides the nominal exchange rate.
- Firms hire labor supplied by households and produce.
- If in good standing in  $t - 1$ , government's decision to default on debt from  $t$ .
- If default, the country switches to bad standing.
- Conditional on good-standing, foreign lenders price country's bonds.
- Conditional on good-standing, households choose debt to be paid in  $t + 1$ .
- If bad standing prevails in Stage-II, loss of  $L(y_t^T)$ .
- Consumption of tradables and non-tradables.





# Chapter 3

## Unstable Monetary Unions - The Role of Expectations and Past Experience

### 3.1 Introduction

Expectations undoubtedly play a key role in modern macroeconomics, but the assumption of rational expectations leaves little room for past experience to influence the formation of expectations. This stands in stark contrast to a growing body of empirical literature that documents the importance of personal experience on the way expectations are formed.<sup>1</sup> This might be especially important in a turbulent economic environment, or one that undergoes a major structural change. A prime example of such an environment is an independent economy that joins a monetary union and, as a result, experiences a change of economic policies and dynamics. Therefore, in this paper I study the role of expectations and past experience in a structurally changing economic environment - concentrating on the example of a small open economy joining a monetary union.

The research question is motivated by the recent Eurozone debt crisis that started with sovereign debt problems in Greece and evolved into encompassing a large part of the currency area. The crisis was one of the most severe post-WWII economic crises in the developed

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<sup>1</sup>The first to point out the relationship between experience and expectations in the context of a rational expectations model were Cukierman and Wachtel (1979). More recently, Malmendier and Nagel (2016) provide intergenerational evidence that personal experience of inflation affects life-time inflation expectations of individuals. Carroll (2003) builds a model where households form expectations based on the news coverage of professional forecasters, who form rational expectations. In the model, agents only rarely pay attention to the news reports, which explains well the stickiness in expectations, and makes expectations dependent on past experience. There is also ample evidence for the importance of past experience on expectations in the context of financial variables and financial decisions; see e.g. Kaustia and Knüpfer (2008), Malmendier and Nagel (2011), and Chiang et al. (2011). Experimental evidence on the importance of past observations for the formation of inflation expectations is provided by a.o. Adam (2007).

world. It also threatened the continued existence of the European Economic and Monetary Union, and undermined European integration in general. Despite the economic and political impact of the crisis and the large body of literature on the topic, there is still little consensus about its causes. Understanding and studying the deep factors behind the crisis is not only important for solving the current problems, but it is also crucial for preventing the build-up of further imbalances that might lead to future crises.

In this paper, I present a theoretical model that is able to capture the importance of past experience for the stability of a country joining a monetary union. More specifically, I introduce informational frictions in the form of learning into a model of a small open economy joining a monetary union and study how those frictions interact with different economic histories. The model reproduces the patterns observed in the Eurozone, where countries with higher inflation experience prior to joining the union accumulate more foreign debt and face a higher risk of economic instability. The paper brings forward inflation histories and inflation expectations as candidates for the deep factors behind the build-up of economic imbalances within the Eurozone. I also investigate the empirical patterns of pre-crisis variables for the Eurozone countries to highlight that pre-euro inflation levels are a good, and so far neglected, country-level predictor of distress during the Eurozone crisis.

I analyze a two-region model of a monetary union, consisting of a small open economy and a large region representing the rest of the union. Private agents in the small open economy are boundedly rational: they know the structure of the economy but do not know the values of the structural parameters. Agents behave as econometricians estimating the parameters of the model based on the data they observe each period. The economies of the two regions in the model are populated by three types of agents: households, firms, and a central bank. The model also features imperfect financial markets, monopolistic competition and nominal frictions.

The main analysis in the paper is a policy experiment in which I analyze the small open economy joining the monetary union of the large region. This is an important structural break for the smaller region, as it permanently fixes its exchange rate and cedes control over its monetary policy to the union-wide central bank. In the simulation exercise, I compare how the economy evolves depending on different initial conditions. In particular, I study how various levels of steady state inflation (prior to joining the union) influence the convergence dynamics in the small region. The main finding of this experiment is that past experience matters. A small open economy with a past steady state inflation level higher than the large region accumulates more foreign debt and faces a higher risk of economic instability. Those unstable dynamics emerge only under learning, but are robust to alterations to the learning algorithm. Under rational expectations past inflation rates have no influence on the current dynamics, and so the economy converges quickly to the new steady state.

The main mechanism of the model works in the following way. The economic dynamics of the small open economy differ substantially depending on whether the economy is independent or part of a monetary union. Therefore, it is a difficult task for private agents in the small open economy to estimate the appropriate dynamic relationship after accession to the union.

When the economy is independent it may adjust to idiosyncratic shocks via three channels: (i) independent monetary policy, in which the central bank follows a Taylor rule and raises interest rates in response to above target inflation; (ii) adjustment of the nominal exchange rate which allows the elimination of any differences in price levels across countries to restore international competitiveness; and finally (iii) the trade channel, through which real activity reacts to movements in the real exchange rate. However, as long as the nominal exchange rate is free to fluctuate, the third channel is inactive.<sup>2</sup> Once the economy joins the monetary union channels (i) and (ii) can no longer be used, as there is a uniform monetary policy in the union, and the exchange rate is permanently fixed. Hence, all the adjustment has to take place through the trade channel, which did not previously play a role.

After joining the union, the agents have to learn about the structural change from observing the behavior of economic variables. If the learning dynamics are too slow or go in the wrong direction then the economy might enter into a region where the flawed expectations lead to instability. The key threats are cross-country heterogeneous inflation expectations combined with a uniform monetary policy. When nominal interest rates are equal across countries, ex ante real interest rates differ with differences in inflation expectations. Specifically, a country with high inflation expectations enjoys low real interest rates. Those low real rates act similarly to expansionary monetary policy, i.e. they stimulate more borrowing and higher spending, which trigger higher inflation. As a result, the country with initially higher inflation expectations experiences higher actual inflation. Thus, the initial beliefs are, at least partially, self-fulfilling, and the convergence of inflation expectations within the union is dampened. A period of prolonged inflation differences will result in substantial price level differences and an accumulation of foreign debt.<sup>3</sup>

The dynamics of the model are the result of two opposing effects: the destabilizing real interest rate effect (described above) and the stabilizing trade effect. The latter relies on higher prices worsening the trade balance and the international competitiveness of the economy, and hence pushing prices down. Under some circumstances the model may display local instability. This might be the case if the effect that higher prices have on trade is relatively weak, while the stimulating effect of the real interest rate is strong. If the real interest rate effect dominates, then higher inflation expectations and inflation are expansionary, and the economy may become unstable. In such an unstable economy, high inflation keeps real interest rates low and drives the accumulation of net foreign debt. The ever-higher foreign debt levels are able to stimulate domestic demand and keep inflation high despite the deteriorating international competitiveness. At the same time, the convergence of beliefs about steady state inflation is not sufficient to stabilize the economy; the agents need to discover the importance of the real exchange rate for economic activity to internalize its stabilizing effect.

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<sup>2</sup>This can be also inferred from the fact that the real exchange rate is not a state variable when the small open economy is independent.

<sup>3</sup>This mechanism is similar to the case of an exchange rate peg when the government lacks full credibility, or needs time to build its credibility, as originally presented by Miller and Sutherland (1993), and by Driffil and Miller (1993). Here, however, the problem is not the credibility of the monetary authority, but rather the information about the new economic environment the country is in.

The results of the model suggest that monetary policy alone might be not enough to stabilize the economies of member countries within a monetary union. The conclusion would hold at least for countries that can be described as small open economies, which in the current context requires the impact of their national variables to be negligible on union-wide monetary policy. Further work on that topic should concentrate on the role of fiscal and macro-prudential policies as additional stabilization tools.

***Related literature*** My paper is related to three strands of the literature. First of all, I contribute to the recent literature using imperfect information and learning to explain the puzzling behavior of inflation dynamics or monetary policy failures. Primiceri (2006) estimates a backward-looking New Keynesian model with learning and shows how learning on the part of the central bank could explain the US Great Inflation and the Volcker disinflation that ended it. Lubik and Matthes (2016) achieve the same effect in a forward-looking framework by introducing learning and real-time data that are burdened with a measurement error. Both papers conceptualize the prolonged rise in inflation as a period of economic instability brought by boundedly rational agents that need to learn the true dynamics of the economy.<sup>4</sup>

In a similar setup, Cogley et al. (2015) consider an economy in which private agents need to learn about the policy rule of the central bank and show that a change by a central bank to a more anti-inflationary monetary policy may lead to temporary indeterminacy. They also derive the optimal policy for achieving disinflation under learning. This policy differs substantially from the optimal policy under rational expectations, as the central bank operating under learning tries to avoid the temporarily explosive dynamics. Similarly, Branch and Evans (2017) consider the possibility of increasing the inflation target in order to reduce the risk of hitting the zero lower bound. In their model, too, the private agents need to learn about the policy change. This may again lead to explosive dynamics, as the agents observing the new policy are not sure whether or not it is still stabilizing the economy.

Closer to the setup of my model is the work on stability under learning in an international economy context, as presented by Bullard and Schaling (2005). They analyze a two-region economy, and study determinacy and learnability issues related to monetary policy. Among other policies, they consider one of the regions fixing its exchange rate and find no risk to stability: the steady state of that economy is determinate and learnable. The role of learning in a monetary union consisting of two regions is also analyzed by Bonam and Goy (2017). They show that learning amplifies the reaction to shocks when agents are myopic. However, even with extreme myopia the economy in their model is still stable under typically considered monetary policy rules.<sup>5</sup>

My paper is the first in this line of literature to explore the consequences of learning and differences in past experience in a monetary union. It is also the first paper to consider the role of expectations and learning as factors in the build-up to the Eurozone crisis.

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<sup>4</sup>The idea of applying adaptive learning to explain the US experience comes from Sargent (1999). Another important study in this line of literature is Sargent et al. (2006).

<sup>5</sup>Learning in macroeconomics has a long tradition. A good overview is provided by Evans and Honkapohja (2009).

The paper is also related to a large body of literature studying the macroeconomic dynamics of a monetary union. There are two seminal contributions in the field. The first is the paper by Benigno (2004), which considers a monetary union as consisting of two regions of equal (or comparable) size. There are many ways in which the basic setup can be extended, and examples include inflation persistence as in Benigno and Lopez-Salido (2006), or imperfect financial markets as in Benigno (2009).

The second seminal contribution in the field is the work by Galí and Monacelli (2008), that builds upon Galí and Monacelli (2005), and considers the monetary union from the perspective of a small open economy. Ferrero (2009) considers the optimal mix of fiscal and monetary policy rules for a currency area. More recent contributions in this line of literature include Eggertsson et al. (2014), Galí and Monacelli (2016), and Hjortsoe (2016). I contribute to this line of literature by highlighting the role of learning and initial conditions on the dynamics within a monetary union.

Finally, there is a very recent strand in the literature that tries to understand the Eurozone crisis and the macroeconomic imbalances that led to it. Estrada and López-Salido (2013) is an early empirical contribution that pointed to the build-up of imbalances and the limited role that relative price developments had on current account balances, providing evidence for the slowness of the effect of the trade channel, and consistent with the limited size of the trade effect in my model under learning.<sup>6</sup>

Piton (2017) takes the divergent behavior of real interest rates in the Eurozone as given and shows that it was a major factor behind the rapid development of the non-tradable sector in the peripheral countries. She finds the Balassa-Samuelson effect to have only a limited impact on the dynamics. Also Gopinath et al. (2017) take the falling real interest rate in the peripheral countries as given and show how they contributed to low technological growth. Their main mechanism works through a financial friction that leads to capital being allocated to the largest, instead of the most productive, firms.

Gilchrist et al. (2014) analyze the effect of international competition and financial frictions on the dynamics of relative prices in a monetary union. They show that, despite large losses in international competitiveness, firms in the periphery might be still inclined to raise prices to limit liquidity constraints that arise due to financial frictions. While at the same time, firms in the core, which already have internationally cheaper products and face no financial constraints, further reduce their prices to undercut their foreign competitors. The model explains the slow adjustment of relative prices between the core and the periphery of the EMU during and after the crisis. De Ferra (2017) considers a model of a monetary union, where agents within the union enjoy a full bail-out promise on all assets issued within the union. His model is able to reproduce the observed current account imbalances.

My paper can be seen as complementary to the existing literature, as it highlights the role of heterogeneous expectations within the union and provides an endogenous mechanism for the divergent behavior of real interest rates in the early years of the monetary union.

***Layout*** The remainder of the paper is structured in the following way. Section 2 inves-

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<sup>6</sup>Also López-Salido and Restoy (2005) find the trade channel to be weak in the Spanish case, despite substantial losses in international competitiveness.

tigates more closely the empirical patterns across Eurozone member countries, and explores the potential candidates for the causes of the crisis. Section 3 presents the model of a small open economy joining a monetary union, and outlines the bounded rationality imposed on the agents. Simulation results of the model replicating the phenomena observed in the EMU are presented in section 4. Section 5 investigates the sensitivity of the results. Section 6 concludes the paper.

## 3.2 In search of the causes of the Eurozone crisis

In a recent attempt to create a common narrative on the causes of the Eurozone crisis Baldwin and Giavazzi (2015) collect the views of eighteen prominent economists. The different arguments made by the invited authors can be summarized by the following list of potential causes: (i) accumulation of public debt; (ii) unregulated expansion of the banking sector; (iii) capital flows and current account imbalances; (iv) loss of competitiveness due to diverging inflation rates; and (v) institutional differences across countries. Those points also capture very well the ongoing policy discussion around the topic.

To understand the importance of these different causes and channels that could be relevant for the Eurozone crisis, in this section I investigate empirical patterns in the Eurozone. Those empirical patterns serve to motivate theoretical research into this topic and set the scene for the quantitative experiments of the model.

Table 3.1 lists several variables connected to the potential causes (i)-(iv) listed above. Instead of reporting the values of those variables, I rank the countries monotonically, starting from the values that are most likely to contribute to a crisis. In this exercise, if a variable is a good predictor of problems during the Eurozone crisis, the countries hit hardest by the crisis should appear at the top of the column.

The variables considered can be divided into three groups: (i) fiscal variables (public debt and deficit as a share of domestic GDP); (ii) banking sector variables (the ratio of bank assets to GDP and the growth rate of that variable); and (iii) variables related to the international position of the country (the current account and inflation). For stock variables I consider the level of 2007, so just before the financial crisis. For flow variables I report the cumulative value over the period 1999-2007, i.e. from the introduction of the euro through till the start of the financial crisis.

The peripheral countries, which are the most severely affected by the Eurozone crisis, are highlighted in each column by the bold and italic font. The pattern that emerges from the table is that the pre-crisis dynamics of the fiscal and banking sector variables are relatively poor indicators of fragility during the crisis, as the countries of interest appear both at the top and bottom of the columns. In contrast, a clear pattern emerges in the last two columns, where persistent current account deficits and persistently high inflation seem to be more indicative of future trouble. Interestingly, the country that fits the pattern least well is Italy. Of the highlighted group, Italy can be considered the country least affected by the crisis, as it is the only country in that group that has not requested any official financial

Table 3.1: Potential causes of the Eurozone crisis. Countries are ranked according to pre-crisis developments in several variables. The countries highlighted by the bold and italic font are the peripheral countries that experienced substantial difficulties during the crisis.

Public finance (% of GDP)		Banking sector (% of GDP)		International competitiveness (% of GDP)	competitiveness (%)
Debt-GDP ratio (2007)	Cumulative budget deficit (1999-2007)	Bank assets (2007)	Growth rate of bank assets (1999-2007)	Cumulative current account (1999-2007)	Cumulative inflation (1999-2007)
<b><i>Greece</i></b>	<b><i>Greece</i></b>	Luxembourg	<b><i>Ireland</i></b>	<b><i>Portugal</i></b>	<b><i>Greece</i></b>
<b><i>Italy</i></b>	<b><i>Portugal</i></b>	<b><i>Ireland</i></b>	Finland	<b><i>Greece</i></b>	<b><i>Ireland</i></b>
Belgium	Finland	Belgium	<b><i>Spain</i></b>	<b><i>Spain</i></b>	<b><i>Spain</i></b>
<b><i>Portugal</i></b>	<b><i>Italy</i></b>	France	<b><i>Italy</i></b>	<b><i>Ireland</i></b>	<b><i>Portugal</i></b>
Eurozone	France	Netherlands	Netherlands	<b><i>Italy</i></b>	Luxembourg
Austria	Austria	Austria	France	Eurozone	Netherlands
France	Germany	Eurozone	Eurozone	France	<b><i>Italy</i></b>
Germany	Eurozone	Germany	Austria	Austria	Eurozone
Netherlands	Belgium	<b><i>Spain</i></b>	<b><i>Greece</i></b>	Germany	Belgium
<b><i>Spain</i></b>	Netherlands	<b><i>Portugal</i></b>	Belgium	Belgium	France
Finland	<b><i>Spain</i></b>	<b><i>Italy</i></b>	<b><i>Portugal</i></b>	Netherlands	Austria
<b><i>Ireland</i></b>	<b><i>Ireland</i></b>	<b><i>Greece</i></b>	Germany	Finland	Finland
Luxembourg	Luxembourg	Finland	Luxembourg	Luxembourg	Germany

support from its EMU partners.

To investigate how robust this pattern is, I also look at the time series for the variables of interest for the original twelve member countries of the European Economic and Monetary Union.<sup>7</sup> The period I will focus on spans the two years prior to the introduction of the euro, i.e. 1997-8, through till shortly before the Global Financial Crisis that later turned into the Eurozone Crisis.<sup>8</sup>

### 3.2.1 Public debt

Figure 3.1 presents the dynamics of public debt in the eurozone member countries. The two panels follow the division of countries into core countries: Austria, Belgium, Finland, France,

<sup>7</sup>I consider an extended group of original member countries, as Greece joined two years later than the other founding countries. Leaving Greece out does not change the patterns considerably.

<sup>8</sup>I limit my analysis to the sample ending in 2008, as the Global Financial Crisis and the Eurozone Crisis could be considered as large international shocks that cannot be captured within my simple framework. One way to think about the crisis years could be as a suddenly binding credit constraint that the agents do not internalize ex ante, but such an analysis is beyond the scope of this paper.



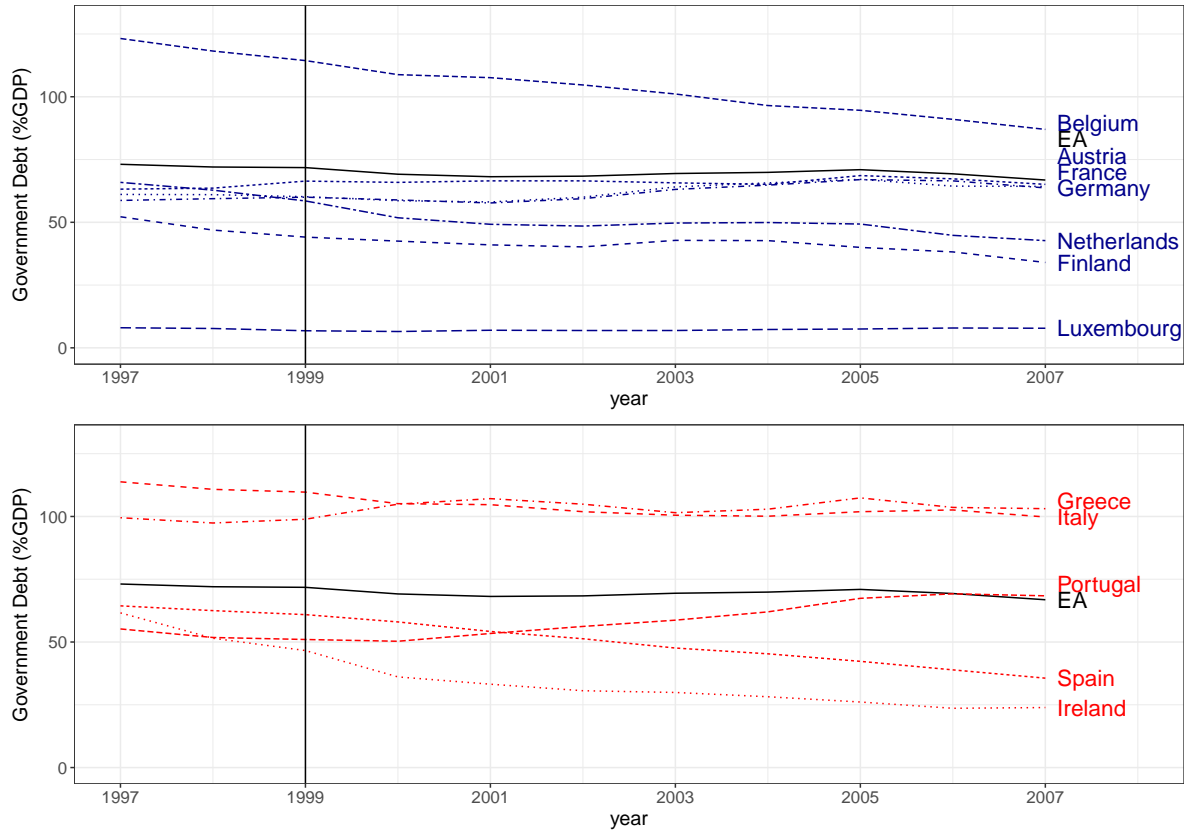


Figure 3.1: Public debt (as % of GDP) of EMU member countries. The upper panel presents the core countries: Austria, Belgium, Finland, France, Germany, Luxembourg, and the Netherlands. The lower panel presents the peripheral countries: Greece, Ireland, Italy, Portugal, and Spain. The solid black line in both panels represents the euro area average. *Source:* own calculations based on Eurostat data.

Germany, Luxembourg, and the Netherlands, and peripheral countries: Greece, Ireland, Portugal, Spain, and Italy. The patterns in the figure reveal several interesting facts. First of all, the debt limit of 60%, set in the Stability and Growth Pact, was violated not only by a large number of countries, but also by the eurozone as a whole, as the aggregate debt to GDP ratio of the union oscillated around 70%. But there is no clear division between the core and the peripheral countries. The three countries with the highest debt levels are Italy, Greece and Belgium, whereas the lowest public debt levels were recorded in Spain, Ireland, and Luxembourg, which means that both the core and the periphery are represented in both extreme groups.

At the same time, there are only four countries that never violated the debt and deficit limits before 2008, the onset of the Global Financial Crisis. Two of them are core countries - Finland and Luxembourg - and two are peripheral countries - Ireland and Spain. This shows clearly that fiscal profligacy was not limited to the peripheral countries, nor was it a

pervasive feature of the group. Therefore, it has no predictive power and does not constitute a good indicator of potential problems within the Eurozone.

### 3.2.2 Banking sector

The second candidate cause of the crisis is the banking sector. Figure 3.2 presents the aggregate level of bank assets in a country as a share of domestic GDP.<sup>9</sup> In the context of the crisis, both the size of the domestic banking sector as its quick growth are mentioned as potential causes for concern.

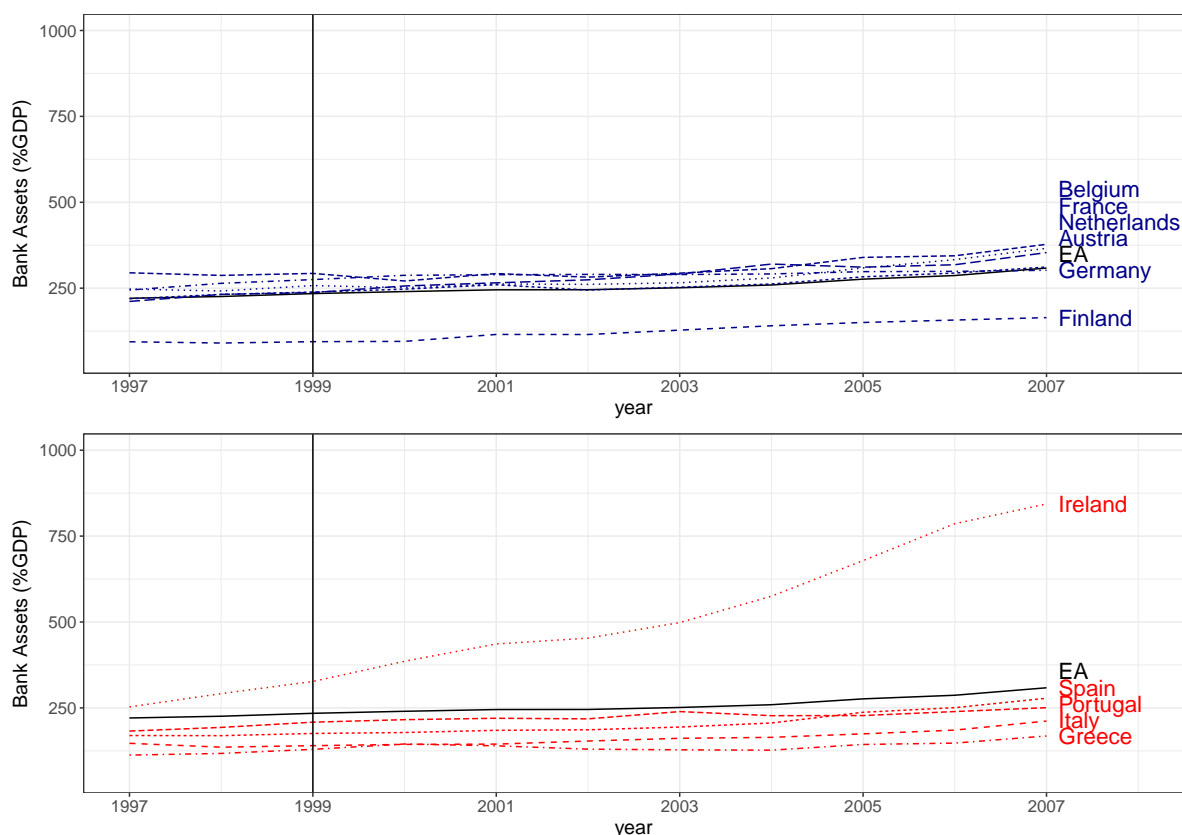


Figure 3.2: Aggregate bank assets (as % of GDP) of EMU member countries. The upper panel presents the core countries: Austria, Belgium, Finland, France, Germany, and the Netherlands. The lower panel presents the peripheral countries: Greece, Ireland, Italy, Portugal, and Spain. The solid black line in both panels represents the euro area average. *Source:* own calculations based on ECB data.

<sup>9</sup>The figures leave out Luxembourg as a clear outlier, whose bank assets-to-GDP ratio stayed constant around 3100%. Including Luxembourg would render the time series for the other countries unreadable.

The general pattern, visible both union-wide as well as at country-level is that the bank-assets-to-GDP ratio grew over the whole period. For the Eurozone as a whole the ratio of bank assets to GDP grew from 220% to 310%. The clear outlier in terms of growth dynamics is Ireland, for which the ratio more than doubled in the period considered. Even so, the level for Ireland is still substantially below the record level of 3100% for Luxembourg. As for the remaining countries, the second fastest growing banking sector was located in Finland. As can be seen in table 3.1, the next two highest positions are occupied by Spain and Italy, but the growth dynamics of their banking sectors are very much comparable with the dynamics in the Netherlands or France. In general, though, most of the lines seem relatively parallel, displaying little differences in pace of growth across a large number of countries in both groups.

A substantial difference between the upper and lower panel seems to be the absolute level of bank assets to GDP, which is lower in the peripheral countries. This actually contradicts the argument that it is a crisis of large banking sectors bringing down the sovereigns: the core countries who were less affected by the crisis tended to have substantially larger banking sectors.

### 3.2.3 Current accounts

Finally, a very important feature of most of the dynamics within the EMU and one of the factors responsible for the Eurozone crisis is the emergence of large current account imbalances within the union, presented in figure 3.3. The two panels of the figure offer a perfect distinction between the core and the periphery countries, where the former record current account surpluses and the latter deficits. At the same time, the current account of the Eurozone as a whole is very close to balanced over period considered. The difference in current account balances was already present in the pre-euro period of 1997-1999, although not as clearly as emerged after the adoption of the euro.

What is striking about figure 3.3 is the persistence of the current account balances. Financial integration could be responsible for the emergence of temporary imbalances, as the elimination of large risk premia (among others the exchange rate premium) might lead to an increase in financial flows. The persistent behavior of current account balances, however, is inconsistent with typical models of a monetary union, or models of an open economy. Any large deviations in current account from zero should converge back to zero or be followed by a period of opposite balances that cancel out the previous ones. Persistent current account deficits translate into an accumulation of net foreign debt, which would be inconsistent with the transversality condition and would imply an explosive path.<sup>10</sup> Therefore, the dynamics portrayed in figure 3.3 pose a challenge to modern international macro models.<sup>11</sup> The insta-

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<sup>10</sup>The effect of large current account deficits could be offset by high growth rates of GDP, as then net foreign debt as share of GDP could remain constant, despite the increase in its absolute value. This was, however, not the case in the peripheral EMU countries.

<sup>11</sup>One explanation for global current account imbalances is provided by Caballero et al. (2008). But even in their model the current account deficits fade away over time, contrary to the dynamics in figure 3.3.

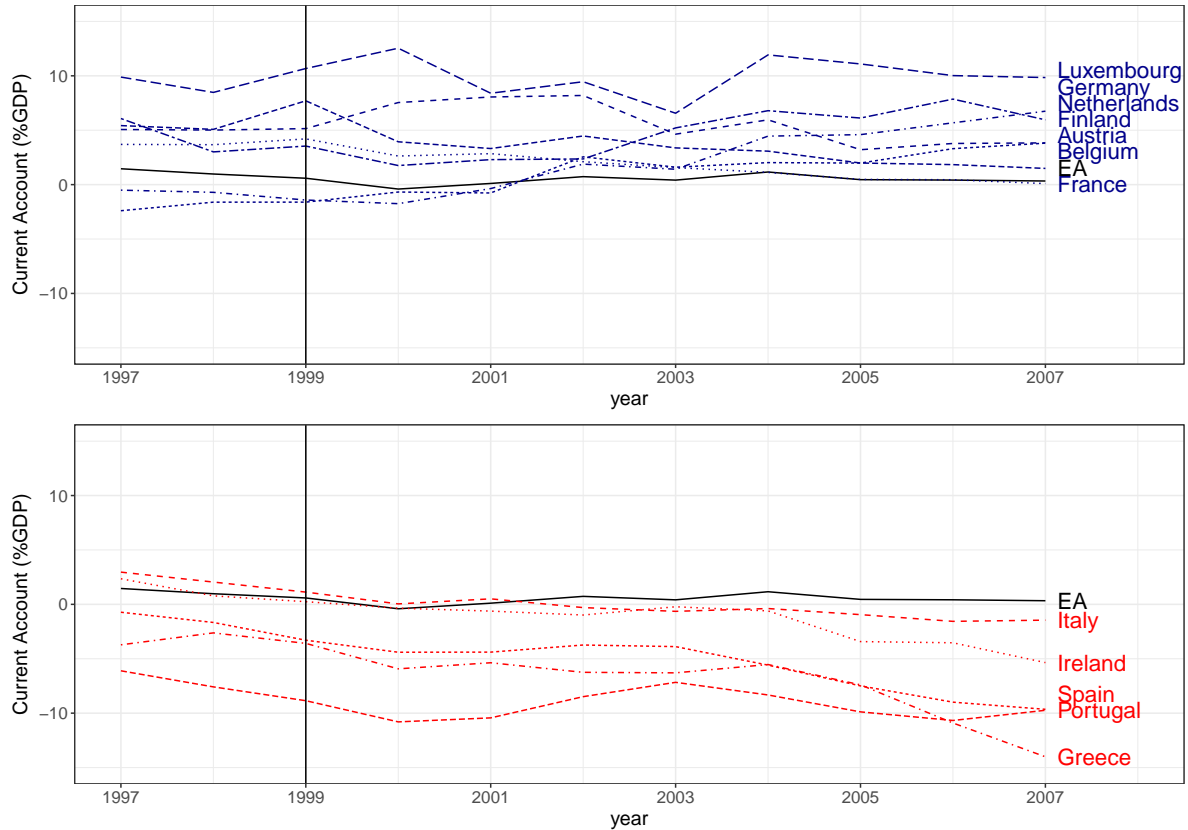


Figure 3.3: Current Account balances (as % of GDP) of EMU member countries. The upper panel presents the core countries: Austria, Belgium, Finland, France, Germany, Luxembourg, and the Netherlands. The lower panel presents the peripheral countries: Greece, Ireland, Italy, Portugal, and Spain. The solid black line in both panels represents the euro area average. *Source:* own calculations based on Eurostat data.

bility of the model, when learning is combined with high inflation experience, is a potential solution to the puzzle.

### 3.2.4 Inflation differences

In the literature inflation features far less prominently as an early indicator of fragility during the Eurozone crisis, but empirically it performs almost equally as well as the current account imbalances. As presented in the upper panel of figure 3.4, the inflation rate of the core countries oscillates around, but close to, the euro area average and, with the exception of the Dutch inflation in 2001, does not exceed 4%. Moreover, most of the actual observations fall below the euro area average and below the ECB's target of 2%.

The dynamics are different in the lower panel of figure 3.4, where almost all observations

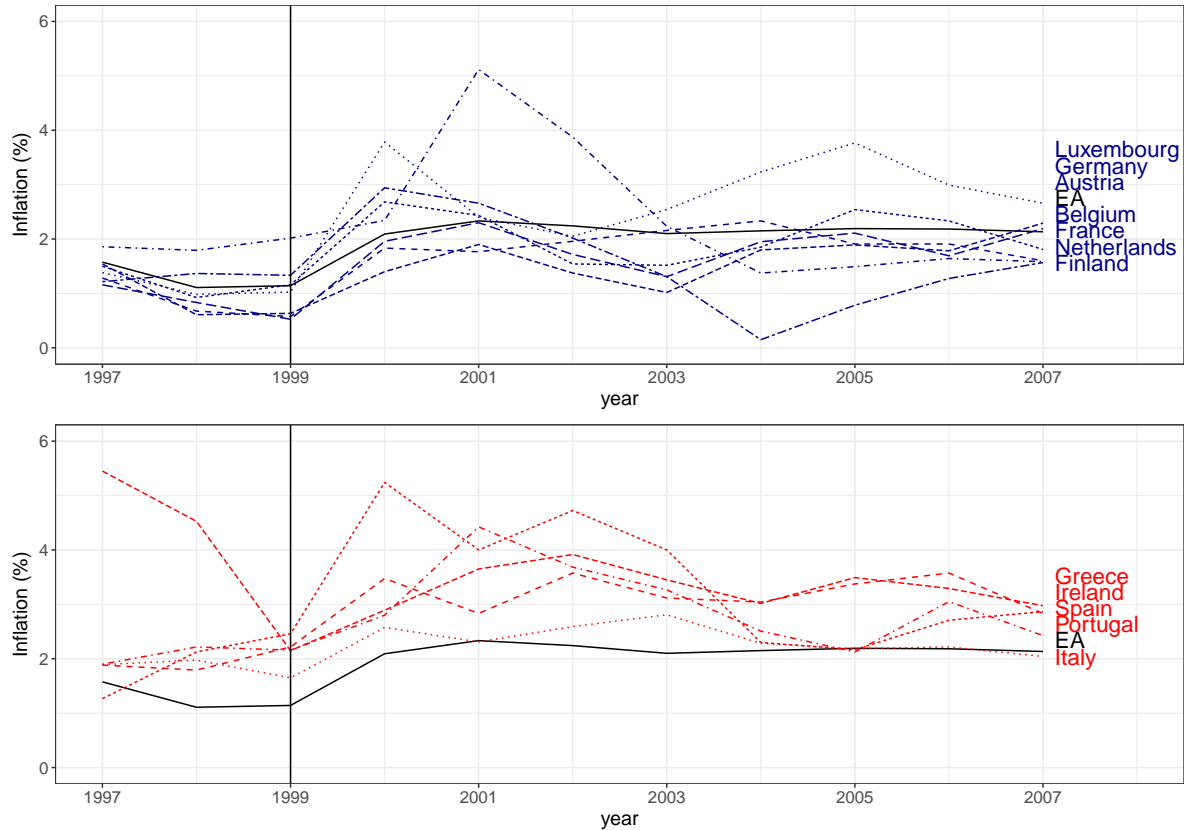


Figure 3.4: Inflation rates of EMU member countries as compared to union-wide inflation. The upper panel presents the core countries: Austria, Belgium, Finland, France, Germany, Luxembourg, and the Netherlands. The lower panel presents the peripheral countries: Greece, Ireland, Italy, Portugal, and Spain. The solid black line in both panels represents the euro area average. *Source:* own calculations based on Eurostat data.

are above the solid line depicting the union-wide inflation rate. Instead of oscillating around the eurozone average, the inflation rates of the peripheral countries seem to persistently exceed the average level, and show little sign of convergence.<sup>12</sup>

The non-convergent behavior of the inflation rates in the periphery constitutes another puzzle that cannot be explained by the existing models of a monetary union. In those models all differences in inflation rates disappear relatively quickly, and the main source of persistence is either price indexation or persistent shocks. In this paper, I address part of

<sup>12</sup>It is interesting to note here that despite the fact that the countries do not display convergence in inflation rates, they did satisfy the *convergence criterion* of price stability. The criterion requires countries applying to join the union to have “an average rate of inflation, observed over a period of one year before the examination, that does not exceed by more than 1.5 percentage points that of, at most, the three best performing Member States in terms of price stability.” The fulfillment of the criterion without actual convergence might be considered suggestive of the weakness of the EMU convergence criteria.

the puzzle and generate inflation persistence even without those features. The source of persistence here are the learning dynamics, which allow current variables to be influenced by past experience via expectations.

The inflation differences observed in the data are not large, which could explain why they have not received much attention. However, as I show in this paper, even relatively small differences might generate trouble.

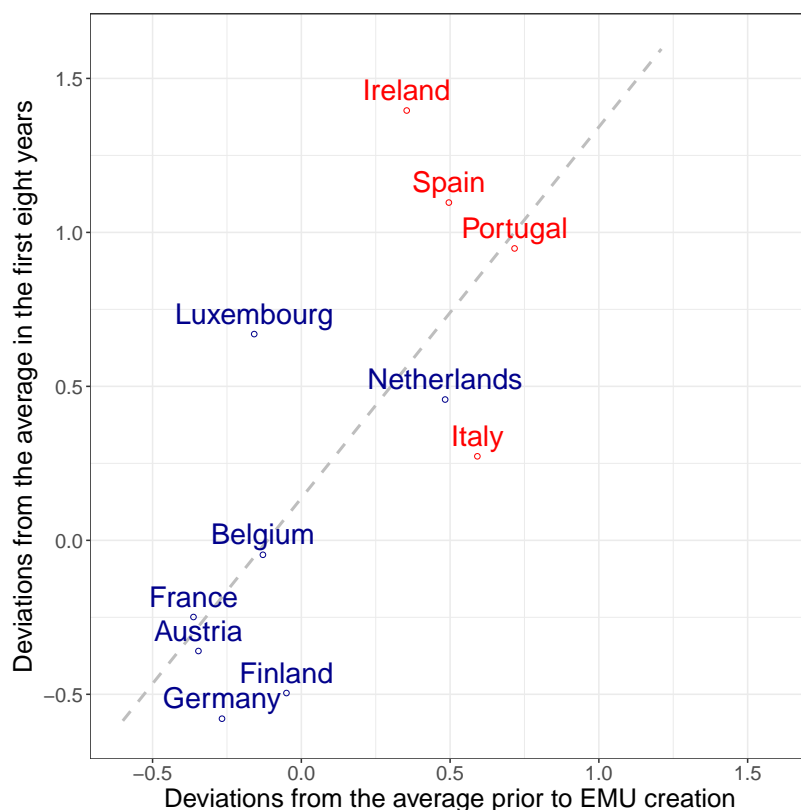


Figure 3.5: Excessive inflation (deviation of domestic inflation rate from the euro area average) in the period 1999-2007 plotted against the excessive inflation in the period 1997-1999. Core countries are printed in blue: Austria, Belgium, Finland, France, Germany, Luxembourg, and the Netherlands. Peripheral countries are plotted in red: Ireland, Italy, Portugal, and Spain. *Source:* own calculations based on Eurostat data.

### 3.2.5 Common cause

The tentative evidence in table 3.1 paired with figures 3.1-3.4 point to current account deficits and excessive inflation as the most successful indicators of the susceptibility of countries to economic distress during the Eurozone crisis. The interesting question is what is the mechanism underlying the development of those two variables and could there be one common

factor explaining both dynamics? The candidate that I put forward in this paper is past inflation experience that influences inflation expectations. Before I introduce the model that explains the novel mechanism, I present some additional empirical patterns that support the plausibility of past experience driving both inflation and the current account balances.

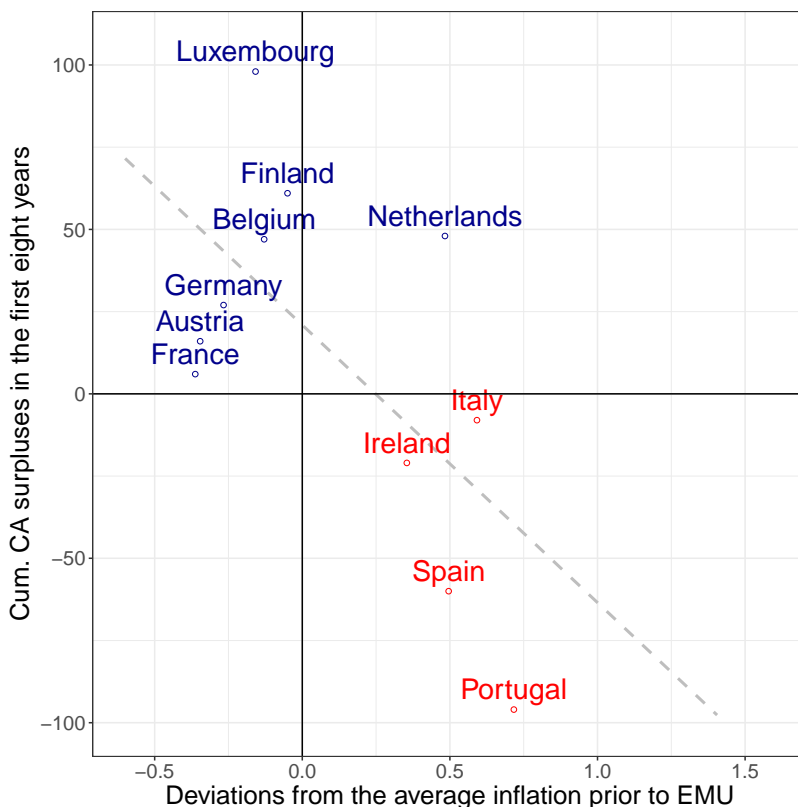


Figure 3.6: Cumulative current account balances (as % of GDP) over the period 1999-2007 plotted against the excessive inflation (deviation of domestic inflation rate from the euro area average) in the period 1997-1999. Core countries are printed in blue: Austria, Belgium, Finland, France, Germany, Luxembourg, and the Netherlands. Peripheral countries are plotted in red: Ireland, Italy, Portugal, and Spain. *Source:* own calculations based on Eurostat data.

Figure 3.5 plots the average deviation of national inflation rates from the union-wide inflation in the period 1999-2007, against the same measure in the period 1997-1999.<sup>13</sup> The trend line in the picture is very close to, but slightly steeper than, the 45-degree line, indicating that inflation deviations from the union-wide inflation are very persistent over time. As mentioned earlier, this high level of persistence cannot be reproduced in standard models.

<sup>13</sup>In figures 3.5 and 3.6 I exclude Greece from the sample, as Greece joined the EMU only in 2001 and the period 1997-1999 is not the appropriate pre-euro convergence period for that country. Including Greece in the scatter-plots would not substantially change the trend lines, nor the general pattern, as Greece's experience is consistent with the remaining peripheral countries.

To complete the picture, figure 3.6 plots the cumulative current account balances in the period 1999-2007 against the average deviation of inflation from the eurozone average in the period 1997-1999. Here again the correlation between the two variables is high but negative. There is also a clear division between countries with low inflation in the pre-euro period and current account surpluses in the pre-crisis period, and countries with high inflation and persistent deficits. Therefore, figures 3.5 and 3.6 highlight that the two successful leading indicators of distress in the Eurozone crisis are highly correlated to the inflation rate prior to joining the monetary union, and could be successfully replaced by it.

### 3.2.6 Inflation expectations

Inflation expectations are a possible channel through which past inflation experience could affect inflation and current account balances, providing an economic interpretation for the empirical patterns described in this section. To investigate the empirical plausibility of the role of inflation expectations in the lack of convergence within the EMU, figure 3.7 presents inflation expectations based on the European Commission's Consumer Survey for the considered period. This survey is conducted among consumers in all EU countries and asks them about their qualitative perception of past and future price changes. The questions are qualitative and I use the methodology of Berk (1999) to get a quantitative measure of inflation expectations.

Both panels of figure 3.7 display a high level of similarity to the inflation series in figure 3.4. In particular, inflation expectations for most of the countries in the lower panel never decrease to the level of the Eurozone average. The only exception being Italy, for which inflation expectations settle at close to and below the Eurozone average after 2001. Perhaps not coincidentally, Italy is also the only peripheral country that did not request financial support during the crisis, which I interpret as a sign of being the least affected in the group.<sup>14</sup>

At the same time, inflation expectations for the whole Eurozone, as well as the expectations in the core countries behave in a very stable manner. In fact, after 2002 none of the inflation expectations in the upper panel exceeds the ECB's target of 2%, indicating that they are well anchored.

In the remainder of the paper I show how limited information of private agents combined with heterogeneity in inflation histories across countries may lead to persistent inflation differences and a build-up of foreign debt, patterns consistent with the ones presented above. The particular information friction I consider is limited knowledge of the structural parameters of the economy. This friction implies that agents form expectations based on past experience. When monetary policy is set union-wide, differences in inflation expectations are not corrected by monetary policy, and are potentially exacerbated.

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<sup>14</sup>During the crisis, of all the peripheral EMU countries Italy also experienced the smallest increase in interest rates on its sovereign bonds.



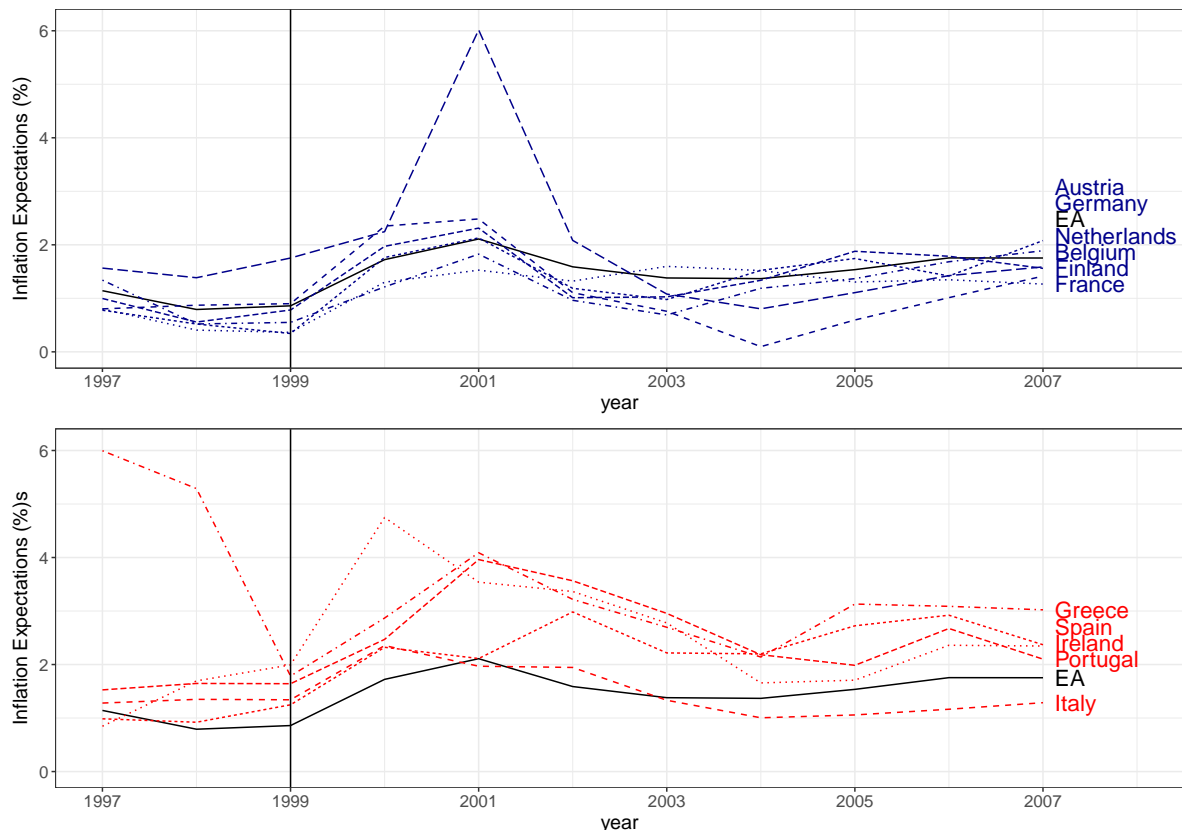


Figure 3.7: Private expectations of inflation rates of EMU member countries as compared to union-wide inflation expectations. The estimates are based on the European Commission's Consumer Survey conducted on a monthly basis. The upper panel presents the core countries: Austria, Belgium, Finland, France, Germany, and the Netherlands. The lower panel presents the peripheral countries: Greece, Ireland, Italy, Portugal, and Spain. The solid black line in both panels represents the euro area average. *Source:* own calculations based on the European Commission's Consumer Survey and Eurostat data.

### 3.3 Monetary Union, Imperfect Information and Inflation Dynamics

In this section I present a two-region model of a small open economy and a large economy, derived as the limit case of a standard two-region new-Keynesian dynamic stochastic general equilibrium model. The model is based on the standard models such as Galí and Monacelli (2005) and De Paoli (2009a). It also allows for a simple extension to consider the case of a country joining a monetary union, as in Galí and Monacelli (2008). One crucial extension that I introduce is the assumption of imperfect financial markets, which follows the work of

Benigno (2009) and De Paoli (2009b).

The model exhibits nominal price rigidities and monopolistic competition to give rise to effective monetary policy and address monetary policy issues. Following the literature, I consider Calvo-type contracts for prices. I also allow for deviations from purchasing power parity in the short run, by assuming some degree of home bias in household preferences.

This model has a dynamic structure and expectations are key for solving it. Under rational expectations agents realize immediately the impact of joining a monetary union for their small open economy, which means that all variables adjust immediately to the new dynamics. I allow for deviations from rational expectations, by assuming that the agents have limited information. In particular, they know the structure of the economy, but they do not know the structural parameters. Instead, they behave as econometricians and learn about the parameters by estimating a model of the economy given past observations.

The learning assumption has far-reaching consequences for the transition from an independent small open economy to being a member of a monetary union. Agents do not know what the union membership will imply for the particular economy that they reside in and how union-wide monetary policy might differ from independent monetary policy. The consequences of this information friction are explored in the next section.

### 3.3.1 The Model Economy

I model the world economy as consisting of two regions, a small open economy and the rest of the world, where initially the latter constitutes the monetary union. The two considered economies are denoted by Home (H) and Foreign (F), respectively. There is a continuum of households of unit mass, of which a fraction  $n$  inhabits region H, and the remaining fraction  $1 - n$  belongs to region F. I analyze the small open economy as a limit case where  $n$  goes to zero. As is standard in the literature, an asterisk will be used to denote variables referring to the F region, whereas I will use no extra notation for domestic variables.

#### Households

A typical household in country H maximizes its lifetime utility, which can be represented by

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_{\tau}, L_{\tau}), \quad (3.1)$$

where  $U(C, L)$  is the one-period utility function. Households derive utility from consumption  $C$ , and disutility from time allocated to work,  $L$ . For what follows I assume that the period utility function takes the form  $U(C, L) = \frac{C^{1-\omega}-1}{1-\omega} - \frac{L^{1+\varphi}}{1+\varphi}$ , where  $\omega$  is the coefficient of relative risk aversion and  $\varphi$  is the inverse Frish elasticity.

The above objective function is maximized subject to a sequence of budget constraints

of the form

$$P_t C_t + \frac{B_{H,t}}{1 + i_t} + \frac{S_t B_{F,t}}{(1 + i_t^*) \Psi \left( \frac{S_t \bar{B}_{F,t}}{P_t} \right)} \leq B_{H,t-1} + S_t B_{F,t-1} + W_t L_t + \Pi_t. \quad (3.2)$$

for all periods  $t$ , where  $P$  is a price index for consumption  $C$ ,  $W$  is the nominal wage earned by households for supplying their labor to domestic firms and  $\Pi$  are profits from domestic firms.

Households have access to both domestic and foreign financial markets, but the markets are incomplete. In particular, there is one instrument for each of the markets, which does not allow households to perfectly share risk across regions.  $B_{H,t}$  and  $B_{F,t}$  denote the net holdings of domestic and foreign bonds at the end of period  $t$ .<sup>15</sup> Since foreign bonds are denominated in foreign currency, they enter the budget constraint multiplied by the nominal exchange rate  $S_t$ . The left hand side of inequality (3.2) thus represents the total of spending on consumption and bond holdings.  $i_t$  and  $i_t^*$  are domestic and foreign net interest rates, where the foreign interest rate is corrected by the function  $\Psi(\cdot)$ , representing the cost of international borrowing.<sup>16</sup> Following De Paoli (2009b) I assume that the cost of international borrowing increases in external debt, hence decreases in net external assets  $\Psi'(\cdot) < 0$ , and that this risk premium is zero when the net external position is zero,  $\Psi(0) = 1$ , which implies a steady state with zero net foreign assets.

The first order conditions of the households are two intertemporal optimality conditions (Euler equations), one for each type of bonds, and a standard intratemporal optimality condition (consumption-leisure trade-off)

$$U_C(C_t, L_t) = \beta E_t \left[ U_C(C_{t+1}, L_{t+1}) (1 + i_t) \frac{P_t}{P_{t+1}} \right], \quad (3.3)$$

$$U_C(C_t, L_t) = \beta E_t \left[ U_C(C_{t+1}, L_{t+1}) (1 + i_t^*) \Psi \left( \frac{S_t B_{F,t}}{P_t} \right) \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right], \quad (3.4)$$

$$U_L(C_t, L_t) = -\frac{W_t}{P_t} U_C(C_t, L_t). \quad (3.5)$$

Equations (3.3) and (3.4) implicitly also define an interest rate parity between home and foreign interest rates. As can be seen from those equations, the two interest rates are allowed to differ due to the borrowing costs faced by domestic households on international markets and to expected changes in the nominal exchange rate.

The consumption good,  $C$ , is a composite good defined by a constant elasticity of sub-

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<sup>15</sup>The net bond holdings could also be negative and represent net liabilities.

<sup>16</sup>The net foreign bond holdings inside the cost function are denoted as  $\bar{B}_F$  to highlight that the borrowing cost depends on the aggregate asset level. This assumption is used to ensure that a single household does not consider its impact on the aggregate level and hence also the borrowing cost, even though in equilibrium  $\bar{B}_F = B_F$  always holds.

stitution aggregator

$$C = \left[ \nu^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \quad (3.6)$$

$C_H$  and  $C_F$  are composite consumption goods produced by the home and foreign economies, respectively. The intratemporal elasticity of substitution between the two composite goods is equal to the parameter  $\theta > 0$ .

Importantly for the limit case of the small open economy, the parameter  $\nu$  is the preference for the home good, and it is defined as  $(1-\nu) = (1-n)\lambda$ , where  $\lambda$  is the degree of openness of the economy,<sup>17</sup> and  $n$  is the size of the home region. When the economy is fully open, i.e.  $\lambda = 1$ , the share of the home good in total consumption is equal to the size of the domestic economy, implying no home bias. In the case of an exclusive home bias,  $\lambda = 0$ , domestic households do not consume the foreign good. Whenever  $\lambda < 1$  households are characterized by a limited degree of home bias.

The home ( $C_H$ ) and foreign ( $C_F$ ) composite consumption goods are CES aggregates of differentiated goods produced in the respective regions. The indices are defined as

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad (3.7)$$

$$C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}. \quad (3.8)$$

Goods indexed from 0 to  $n$  are produced in region H, and from  $n$  to 1 in region F. The elasticity of substitution between differentiated products from the same region is assumed to be constant across regions and equal to  $\sigma > 1$ .

Households in region F consume a composite consumption good, which is defined similar to equation 3.6, as

$$C^* = \left[ (\nu^*)^{\frac{1}{\theta}} (C_H^*)^{\frac{\theta-1}{\theta}} + (1-\nu^*)^{\frac{1}{\theta}} (C_F^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (3.9)$$

where  $\nu^* = n\lambda$ . Equations (3.7)-(3.8) have their exact equivalents for the F region.<sup>18</sup>

Let  $p(z)$  be the price of a differentiated good  $c(z)$  in Home. Then from the intratemporal optimization constraint of the domestic household I obtain the price indices for the region-

<sup>17</sup>The openness of the economy is defined only in terms of trade, as financial markets are fully open.

<sup>18</sup>For the sake of brevity the exposition of the problem of foreign households is postponed to Appendix A, except for equations that are key for the presentation of the domestic problem.

specific composite goods

$$P_H = \left[ \frac{1}{n} \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (3.10)$$

$$P_F = \left[ \frac{1}{1-n} \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (3.11)$$

$$P = [\nu P_H^{1-\theta} + (1-\nu) P_F^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (3.12)$$

With equivalent results for the foreign region presented in equations (3.55)-(3.57).

Furthermore, I assume that the law of one price holds at the level of the differentiated goods, hence

$$p(z) = Sp^*(z), \quad (3.13)$$

where  $S$  is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. The equality is assumed to hold for all  $z \in [0, 1]$ .

The above condition, together with the definition of price indices (3.10)-(3.11) and (3.55)-(3.56), implies that the law of one price holds also at the level of origin-specific composite goods, i.e.  $P_H = SP_H^*$  and  $P_F = SP_F^*$ . However, due to the home bias, the general price levels in the two regions do not need to be equal. To keep track of the relative price level, let us define the real exchange rate as

$$Q \equiv \frac{SP^*}{P}. \quad (3.14)$$

Combining the intratemporal optimization problems of domestic and foreign households with the definition of the real exchange rate, also allows us to write down the total demand equations for differentiated goods produced in the two regions as a sum of domestic and foreign demands

$$y^d(h) = \left( \frac{p(h)}{P_H} \right)^{-\sigma} \left[ \left( \frac{P_H}{P} \right)^{-\theta} \left( \nu C + \frac{\nu^*(1-n)}{n} Q^\theta C^* \right) \right], \quad (3.15)$$

$$y^d(f) = \left( \frac{p(f)}{P_F} \right)^{-\sigma} \left[ \left( \frac{P_F}{P} \right)^{-\theta} \left( \frac{(1-\nu)n}{1-n} C + (1-\nu^*) Q^\theta C^* \right) \right], \quad (3.16)$$

where  $h \in [0, n]$  is used for goods produced in H and  $f \in (n, 1]$  is used for foreign produced goods. The superscript  $d$  is used to indicate that the above equations define the demand side of the economy.

Finally, following De Paoli (2009b), I consider the special case of region H being a small

open economy, by taking the limit for  $n \rightarrow 0$ . This reduces the preference parameters to

$$\lim_{n \rightarrow 0} \nu = 1 - \lambda \quad \text{and} \quad \lim_{n \rightarrow 0} \nu^* = 0. \quad (3.17)$$

## Firms

The problem of the firm is typical in the literature, as presented by e.g. Galí and Monacelli (2005). Firms produce a differentiated good using a linear technology represented by the production function

$$y_t(z) = \varepsilon_t N_t(z), \quad (3.18)$$

where firm-specific labor is the only production input, and  $\varepsilon_t$  is a time-varying region-wide technology parameter.<sup>19</sup>

This specification of the production technology implies nominal marginal cost (expressed in domestic producer prices) to be common across all domestic firms and equal to

$$MC_t^n = \frac{W_t}{\varepsilon_t}. \quad (3.19)$$

Firms face a nominal friction in the form of Calvo-type partial price adjustment. Each period a fraction  $1 - \alpha$  of randomly selected firms is allowed to change prices. The remaining fraction  $\alpha$  retains prices from the previous period. Therefore, the optimal price (chosen by the firms allowed to reset their prices) is chosen by optimizing a discounted stream of expected profits

$$E_t \left\{ \sum_{\tau=t}^{\infty} \alpha^{\tau-t} \Lambda_{t,\tau} y_{\tau}(h)^d \left[ \tilde{P}_{H,t} - MC_{\tau}^n \right] \right\} \quad (3.20)$$

subject to the demand equation (3.15) with  $p_{\tau}(h) = \tilde{P}_{H,t}$ , for periods  $\tau = t, t+1, \dots$ . Since firms are owned by households, the profits are discounted using the stochastic discount factor of the households

$$\Lambda_{t,\tau} = \beta^{\tau-t} E_t \left[ \frac{U_C(C_t, L_t)}{U_C(C_{\tau}, L_{\tau})} \frac{P_t}{P_{\tau}} \right].$$

The resulting aggregate price dynamics follow

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) \left( \tilde{P}_{H,t} \right)^{1-\sigma}, \quad (3.21)$$

where  $\tilde{P}_{H,t}$  is the optimal price chosen in period  $t$ .

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<sup>19</sup>The firm sectors in the two regions are symmetric. Therefore, I limit my presentation to the domestic economy. All the equations presented also hold for the foreign economy.

## Monetary policy

Monetary policy is conducted by an independent central bank of the small open economy that pursues price stability. To achieve its policy goal the central bank sets the domestic interest rate according to a policy rule that takes the standard form of a Taylor rule. In particular it can be described as

$$i_t = \bar{i} + \phi_\pi (\pi_t - \bar{\pi}) + m_t, \quad (3.22)$$

where  $\bar{i}$  is the natural (or steady state) nominal interest rate,  $\phi_\pi$  is the response parameter, and  $\bar{\pi}$  is the inflation target. Monetary policy is also subject to IID shocks, represented here by  $m$ .

The central bank of the F region follows a similar rule

$$i_t^* = \bar{i}^* + \phi_\pi^* (\pi_t^* - \bar{\pi}^*) + m_t^*. \quad (3.23)$$

The two central banks set the interest rates independently, concentrating solely on the variables of their region. Differences in the two interest rates are accommodated by either expected future changes in the exchange rate, or by the risk premium that Home households pay on foreign debt, as can be seen by comparing equations (3.3) and (3.4).<sup>20</sup>

The relationship between the two interest rates changes once the country joins the monetary union. Then, the foreign (union) central bank still sets the interest rate according to equation (3.23), but the domestic central bank cedes all its powers to the foreign one, and the domestic interest rate becomes simply the foreign (union-wide) interest rate adjusted by the borrowing cost.

## Equilibrium dynamics

The equilibrium in this model is a sequence of prices and quantities such that the optimality conditions for households and firms in the two regions hold, labor markets and the domestic (home) asset market clear at a regional level, goods markets in both regions and the foreign asset market all clear on an international level, while monetary policies in the two regions follow the specified policy rules.<sup>21</sup>

I approximate the equilibrium dynamics of the model around the symmetric steady state, by a system of log-linearized equations.<sup>22</sup>

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa [(\omega + \phi(1 - \lambda))\hat{y}_t - \omega\lambda\hat{y}_t^* - (1 - \lambda)(1 + \phi)\varepsilon_t - (\omega\gamma - \lambda)\hat{q}_t] \quad (3.24)$$

---

<sup>20</sup>The uncovered interest rate parity condition is defined explicitly in equation (3.28).

<sup>21</sup>Some of those conditions, especially those concerning the foreign asset market can be found in Appendix A. A formal equilibrium definition with a full list of equilibrium conditions is available upon request.

<sup>22</sup>The derivation of the equilibrium conditions is available upon request.

$$E_t \Delta \hat{y}_{t+1} = \lambda E_t \Delta \hat{y}_{t+1}^* + \frac{1-\lambda}{\omega} [\hat{i}_t - E_t \hat{\pi}_{t+1}] + \frac{\lambda + \gamma \omega}{\omega} E_t \Delta \hat{q}_{t+1}, \quad (3.25)$$

$$\beta \hat{b}_t = \hat{b}_{t-1} - \frac{\lambda}{1-\lambda} (\hat{y}_t - \hat{y}_t^*) + \frac{\gamma + \lambda}{1-\lambda} \hat{q}_t, \quad (3.26)$$

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + m_t, \quad (3.27)$$

$$\hat{i}_t = \hat{i}_t^* - \delta \hat{b}_t + \frac{1}{1-\lambda} E_t \Delta \hat{q}_{t+1} + E_t [\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*], \quad (3.28)$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa [(\omega + \phi) \hat{y}_t^* - (1 + \phi) \varepsilon_t^*], \quad (3.29)$$

$$E_t \Delta \hat{y}_{t+1}^* = \frac{1}{\omega} [\hat{i}_t^* - E_t \hat{\pi}_{t+1}^*], \quad (3.30)$$

$$\hat{i}_t^* = \phi_\pi^* \hat{\pi}_t^* + m_t^*. \quad (3.31)$$

Variables with a “” represent log-deviations from steady state.<sup>23</sup>

Equations (3.24) and (3.25) are the small open economy versions of the NK Philips curve and the NK IS curve. Equation (3.24) features two newly defined parameters,  $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1-\lambda)}$  and  $\gamma \equiv \theta [1 - (1-\lambda) + \frac{\lambda}{1-\lambda}]$ . Similarly, equations (3.29) and (3.30) are the NK Philips curve and the NK IS curve for region F. Since I consider the extreme version of a two region model, where region H is a small open economy, the dynamics in region F are equivalent to a closed economy.

The log-linearized versions of the Taylor rules for each of the regions is presented in equations (3.27) and (3.31). Equation (3.26) is the budget constraint of a household in region H. This equation is important for equilibrium dynamics, as with incomplete financial markets the dynamics of net foreign assets have non-trivial consequences. Finally, since domestic households have access to both domestic and foreign financial markets, the uncovered interest parity condition between the two interest rates that they face in those markets needs to hold. This is represented by equation (3.28).

### 3.3.2 The Monetary Union

When the small open economy, represented in the model by region H, creates a monetary union with region F it experiences several structural changes. First of all, it adopts the currency of the union, which permanently fixes the nominal exchange rate between the two regions,  $S_t \equiv 1$ . The small open economy also subjects its monetary policy to the central bank of region F, which becomes now the union-wide central bank.

The first change implies that the exchange rate  $S_t$  is constant over time and equal to 1. This eliminates an important degree of freedom for international adjustments, as previously any loss of competitiveness could have been compensated for with a depreciation that would equate price levels across countries. Now, price level differences need to be corrected for by internal devaluations, i.e. a fall of the domestic price level.

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<sup>23</sup>There are two exceptions to this notation rule. The first is  $\hat{b}_t = \frac{B_t - \bar{B}}{\bar{Y}}$ , where  $B_t \equiv \frac{S_t B_{F,t}}{P_t}$ , and variables with a bar represent steady state values. The second exception is  $\hat{i} \equiv i_t - \bar{i}$ .



The loss of independent monetary policy implies that the interest rate in country H is set equal to the union-wide rate,  $i_t^*$ , corrected with the country premium,  $\Psi(\cdot)$ . One could imagine that upon joining the monetary union, agents in the country should be able to borrow at the union-wide rate without facing a country-specific premium. In fact, many EMU countries observed substantial drops in country premia after accession. Here, I keep the country premium, as it is necessary to close the model and pin down a steady-state asset portfolio.<sup>24</sup> It also makes it easier for agents to learn the new parameters, as they are closer to the pre-accession values. Therefore, if anything, this assumption should bias the simulation results in the next section towards stability.

The loss of independent monetary policy eliminates another potential channel for correcting inflation differences across countries, as the union-wide interest rate is set in reaction to the union-wide inflation rate. The weight of country H in this union-wide average is equal to zero, implying that the central bank does not adjust its interest rate as long as inflation in the rest of the union stays close to its target, even if inflation in country H deviates substantially from the target.

The equilibrium dynamics of the model after the accession of country H to the monetary union can be described by a system similar to the log-linearized system (3.24)-(3.31).

$$\begin{aligned}\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa [(\omega + \phi(1 - \lambda))\hat{y}_t - \omega \lambda \hat{y}_t^* - (1 - \lambda)(1 + \phi)\varepsilon_t - (\omega\gamma - \lambda)\hat{q}_t], \\ E_t \Delta \hat{y}_{t+1} &= \lambda E_t \Delta \hat{y}_{t+1}^* + \frac{1 - \lambda}{\omega} [\hat{i}_t - E_t \hat{\pi}_{t+1}] + \frac{\lambda + \gamma\omega}{\omega} E_t \Delta \hat{q}_{t+1},\end{aligned}\tag{3.33}$$

$$\beta \hat{b}_t = \hat{b}_{t-1} - \frac{\lambda}{1 - \lambda} (\hat{y}_t - \hat{y}_t^*) + \frac{\gamma + \lambda}{1 - \lambda} \hat{q}_t,\tag{3.34}$$

$$\hat{i}_t = \hat{i}_t^* - \delta \hat{b}_t,\tag{3.35}$$

$$\hat{q}_t = \hat{q}_{t-1} + (1 - \lambda) [\hat{\pi}_t - \hat{\pi}_t^*],\tag{3.36}$$

$$\pi_t^* = \beta E_t \pi_{t+1}^* + \kappa [(\omega + \phi)\hat{y}_t^* - (1 + \phi)\varepsilon_t^*],\tag{3.37}$$

$$E_t \Delta \hat{y}_{t+1}^* = \frac{1}{\omega} [\hat{i}_t^* - E_t \hat{\pi}_{t+1}^*],\tag{3.38}$$

$$\hat{i}_t^* = \phi_\pi^* \hat{\pi}_t^* + m_t^*.\tag{3.39}$$

The main difference between the two systems is the fact that the domestic Taylor rule, equation (3.27), and the no-arbitrage condition, equation (3.28) are replaced by equations (3.35) and (3.36). The former is the union-wide monetary policy transmitted to region H, and the latter describes the dynamics of the real exchange rate. Those dynamics are now important as they represent differences in price levels across the two regions, which, in turn, define the competitiveness of the domestic economy. Previously the real exchange rate could adjust to divergent price movements through the nominal exchange rate. After monetary integration this channel is shut down because of the common currency. Hence, persistent inflation differences may have long-lasting consequences.

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<sup>24</sup>For a discussion of other possibilities of closing a SOE model, see Schmitt-Grohe and Uribe (2003).

### 3.3.3 Learning

Agents in the economy are assumed to have limited information. They know the structure of the economy, but do not know the value of the structural parameters. Therefore, they behave as econometricians, using their observations of the economy to estimate the parameters.<sup>25</sup>

The model economy presented in the previous subsections can be represented by a system of linear equations

$$AX_t = B\tilde{\mathbb{E}}_t X_{t+1} + CX_{t-1} + Du_t, \quad (3.40)$$

where  $X_t$  is the vector of state variables,  $u_t$  are shocks with an expected value,  $\tilde{\mathbb{E}}_t u_{t+1} = 0$ , and  $A$ ,  $B$ ,  $C$  and  $D$  are matrices of parameters. The expectations operator  $\tilde{\mathbb{E}}$  has a tilde, indicating that expectations might differ from rational expectations.

Under rational expectations, the law of motion for this economy can be represented by the reduced form system

$$X_t = FX_{t-1} + Gu_t, \quad (3.41)$$

where  $F$  and  $G$  are parameter matrices derived as functions of the original parameters  $A$ ,  $B$ ,  $C$  and  $D$ .

However, in my model agents do not know the parameter matrices. Instead they form their own view of the economy that can be represented by the system

$$X_t = F_t X_{t-1} + G_t \tilde{u}_t, \quad (3.42)$$

where the parameter matrices have time subscripts indicating that they may change over time as agents update their estimates. This represents only the change in the perception of the agents, as the true parameters of the economy are constant over time. Also, the vector of shocks is now  $\tilde{u}_t$ , representing the perceived shocks that might differ from the actual shocks.

This reduced-form VAR model is the agents' Perceived Law of Motion (PLM). They use the PLM to form expectations about next period variables.

To get the Actual Law of Motion (ALM) for the economy we need to substitute the PLM for the expectations in equation (3.40), hence

$$AX_t = B \left[ F_t X_t + G_t \tilde{\mathbb{E}}_t \tilde{u}_{t+1} \right] + CX_{t-1} + Du_t, \quad (3.43)$$

Reorganizing the above and remembering that the expected value of the perceived future shocks is zero, we obtain the ALM

$$X_t = (A - BF_t)^{-1} (CX_{t-1} + Du_t). \quad (3.44)$$

---

<sup>25</sup>Evans and Honkapohja (2001) is a good reference for a more detailed treatment of adaptive learning in macroeconomic models. The exposition in this section follows Carceles-Poveda and Giannitsarou (2007).

Hence, the ALM depends not only on the structural parameters of the economy, but also on the agents' perception of those parameters, which may change over time. Hence, also the ALM varies over time.

## Timing

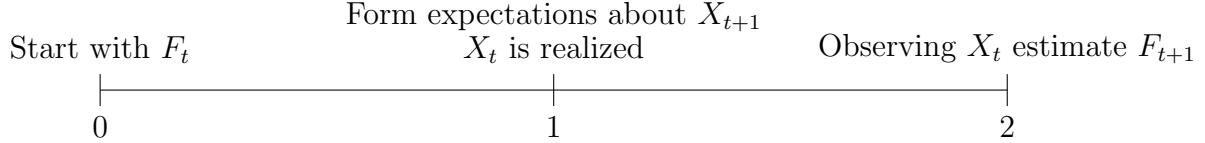


Figure 3.8: Timeline of the actions for period  $t$

The timeline of the actions within one period is presented in figure 3.8. The private agents start the period with an estimate of the structural parameters  $F_t$  based on the information available in the previous period. Then using this estimate, they form expectations, which translate into current period choices. At the end of the period, the agents update their estimate to  $F_{t+1}$  using all information available in period  $t$ .

## Constant Gain Learning

The agents in the economy update their beliefs about the parameters using a recursive formula of the form

$$R_t = R_{t-1} + g_t (X_{t-1} X_{t-1}^T - R_{t-1}) \quad (3.45)$$

$$F_{t+1} = F_t + g_t R_t^{-1} X_{t-1} (X_t - F_t X_{t-1}), \quad (3.46)$$

where  $R_t$  is an auxiliary variable, and  $g_t$  is the gain parameter defining the speed of learning. The above system boils down to a standard recursive least squares estimator for a gain parameter  $g_t = 1/t$ .

Instead, the agents in our economy use a popular alternative to RLS, namely the constant gain version of RLS, where  $g_t = g$ . This assumption implies that the agents put a higher weight on more recent observations. This assumption can be motivated by the setup of the model, where agents have to learn the new parameters after their country joins a monetary union. The structural break implies that past observations might be misleading, as they were realized under the previous regime.

## 3.4 Inflation convergence in a monetary union

To investigate how past differences in inflation rates affect the stability of a small open economy joining a monetary union, I calibrate the model presented in the previous section and run simulations for different initial conditions. Here, initial conditions refer to the steady state inflation rate prior to joining the union. Keeping everything besides initial conditions fixed between different scenarios allows me to analyze the impact of past inflation experiences on the dynamics of the model. I also compare the behavior of the model under learning and rational expectations.

### 3.4.1 Calibration

The model cannot be solved analytically and I solve the log-linearized version of the model numerically. The choice of the calibrated parameters follows closely the macroeconomic literature on small open economies. In particular, the calibration is similar to that of De Paoli (2009b), who analyzes a framework very similar to my basic model.

The model is calibrated at quarterly frequency, and the values of the calibrated parameters are reported in table 3.2. I set the subjective discount factor equal to 0.99, which implies an annual risk-free interest rate of approximately 4%. I choose the coefficient of relative risk aversion  $\omega = 2$ ; as is commonly used in the macroeconomics literature. The set of parameters of the utility function of the household is completed with an inverse Frish elasticity of 2, which is between the value of 0.47 typically chosen by the literature following Rotemberg and Woodford (1997), and the higher value of 3, chosen in the context of a small open economy by a.o. Galí and Monacelli (2005).

For the openness to trade parameter  $\lambda$ , which in the limit case of a small open economy determines the ratio of import to output, I select the value 0.25. This is in line with De Paoli (2009b), but lower than the value of 0.4 chosen by Galí and Monacelli (2005). More importantly, however, the selected value corresponds well with the trade patterns of the peripheral eurozone countries for which the average level of intra-EU imports varies between 12% and 26% of domestic GDP, and the value of total imports is in the range of 22% to 33% of GDP.

In the choice of parameters for the intratemporal elasticities of substitution I follow De Paoli (2009b), which relies on previous literature. I set the elasticity of substitution between domestic and foreign goods,  $\theta = 1.5$ , and the elasticity of substitution between differentiated goods from the same region,  $\sigma = 10$ . The latter corresponds to a steady state level mark-up of approximately 11%. I set the average length of price contracts to three quarters, which is equivalent to setting the share of firms that cannot adjust their prices within a given period to  $\alpha = 0.66$ .

Both central banks follow a Taylor rule, and I set the reaction coefficient to deviations from inflation to 1.5, the typical value in the literature. For the sensitivity of the interest rate, I follow the literature on imperfect financial markets in international macroeconomics. In particular, I assume  $\delta = 0.01$ , as is done by Benigno (2009) and De Paoli (2009b).

The model features four exogenous shocks: productivity shocks in both regions,  $\varepsilon$  and

Table 3.2: Calibrated parameters of the model

Parameter	Value	Description
$\beta$	0.99	Discount factor
$\omega$	2	CRRA coefficient
$\varphi$	2	Inverse Frish elasticity
$\lambda$	0.25	Openness to trade
$\theta$	1.5	Intratemporal elasticity of substitution
$\sigma$	10	Elasticity of substitution between the differentiated products
$\delta$	0.01	Sensitivity of borrowing costs to the level of net foreign assets
$\alpha$	0.66	Calvo price stickiness parameter
$\phi_\pi$	1.5	Coefficient on inflation in the SOE Taylor rule
$\phi_\pi^*$	1.5	Coefficient on inflation in the union-wide Taylor rule
$\text{std}(\varepsilon)$	0.013	Standard deviation of the technological shock
$\text{std}(\varepsilon)$	0.013	Standard deviation of the technological shock
$\text{std}(m)$	0.001	Standard deviation of the monetary policy shock
$\text{std}(m^*)$	0.001	Standard deviation of the monetary policy shock
$g$	0.025	Gain parameter for the learning algorithm

$\varepsilon^*$ ; and monetary policy shocks in both regions,  $m$  and  $m^*$ . I assume that the shocks are independent of each other and follow a normal distribution. I also shut down the external persistence of the model by assuming the autocorrelation of the shocks to be zero. This allows me to assign any remaining persistence to the learning dynamics. The values for the standard deviations of the shocks follow De Paoli (2009b).<sup>26</sup>

In the basic setup I assume that private agents in region H use the constant gain least squares algorithm for learning. It is the most popular algorithm in the literature, and it allows the agents to discount past observations more heavily. The latter feature is especially attractive for a model with a structural break, such as the accession to a monetary union.

The algorithm requires the choice of a constant gain parameter.<sup>27</sup> Milani (2007) is one of the first to estimate a DSGE model with learning, and the mean of his posterior distribution is 0.018. Other studies on US data find similar results - Milani (2008) obtains an estimate of 0.02 and Milani (2011) of 0.0196. One of the few estimates based on Eurozone data is Milani (2009), whose estimates for the union as a whole are in the range of 0.004-0.008. Finally, Berardi and Galimberti (2017) evaluate gain parameters for a broad class of models

<sup>26</sup>The patterns presented in the next section - i.e. the postponed convergence of inflation and the accumulation of net external debt that persists even after the convergence of inflation, as well as the increased risk of instability - are insensitive to scaling the variance of shocks. The variance of the shocks should not be too small, however, as the shocks are important for the private agents estimating the reduced form of the model.

<sup>27</sup>The gain parameter is not a structural parameter and hence is specific to the considered model. Nevertheless, I treat the existing literature as educative about possible values used in studies with similar models.

using both actual and survey data and find values in the range of 0.01-0.035 to perform the best for inflation rates. I calibrate the parameter to 0.025, which is in the upper range of the aforementioned studies. In the next section I analyze the sensitivity of the results with respect to the gain parameter.

### 3.4.2 Simulations

In the simulation exercise I consider two basic setups. In both cases a small open economy joins a monetary union. In the first setup the economy operates under the same steady state inflation rate as the rest of the union, already prior to joining the union. Whereas in the second setup, it operates under a higher steady-state inflation rate prior to the integration. In both cases, however, the post-entry steady-state inflation rate in the small open economy is equal to the one in the monetary union.

Since past inflation rates do not influence current dynamics under rational expectations, the behavior of the economy is exactly the same in the two setups when agents are fully rational. The dynamics might differ, though, under learning.

I run 2000 simulations for each of the two setups, with a set of shocks fixed across setups, but differing across simulations. In each of the simulations the economy first operates for 60 periods (fifteen years) as an independent small open economy. The initial training period is necessary to generate initial conditions for the learning exercise. I vary only the steady-state inflation level in the training period, and analyze how those differences influence the learning process, and how the dynamics under learning differ from the dynamics under rational expectations.

Figures 3.9 and 3.10 show the median dynamics of inflation and current account balances, respectively, in the first 100 periods after union-accession over 2000 simulations. They also present the simulation-based confidence intervals, which are calculated as the *15th* and *85th* percentile of the simulations in each period. The solid gray lines in both panels of figure 3.9 show the behavior of the median simulated inflation rate under rational expectations, whereas the solid black line represents the median dynamics under learning. The left panel represents the case of no past inflation differences, and the right panel shows the case of the past inflation target being higher by one percentage point. As expected, under rational expectations, the behavior of inflation differs only in the first period, and then stays the same for both setups, as past inflation experience is inconsequential for economic dynamics.

This is no longer the case under learning, plotted with the solid black line. For the setup with no past inflation differences, inflation dynamics under learning are very close to the dynamics under rational expectations (left panel). They differ substantially, though, in the right panel, i.e. in the setup with higher past inflation. In this setup, inflation first jumps up, to converge slowly in the first two years of union membership. The period of higher inflation seems short, but it should be noted that the model has no external sources of persistence - all shocks are independent across time and there is no indexation in the model - which means that all the persistence comes from the learning dynamics. Importantly,

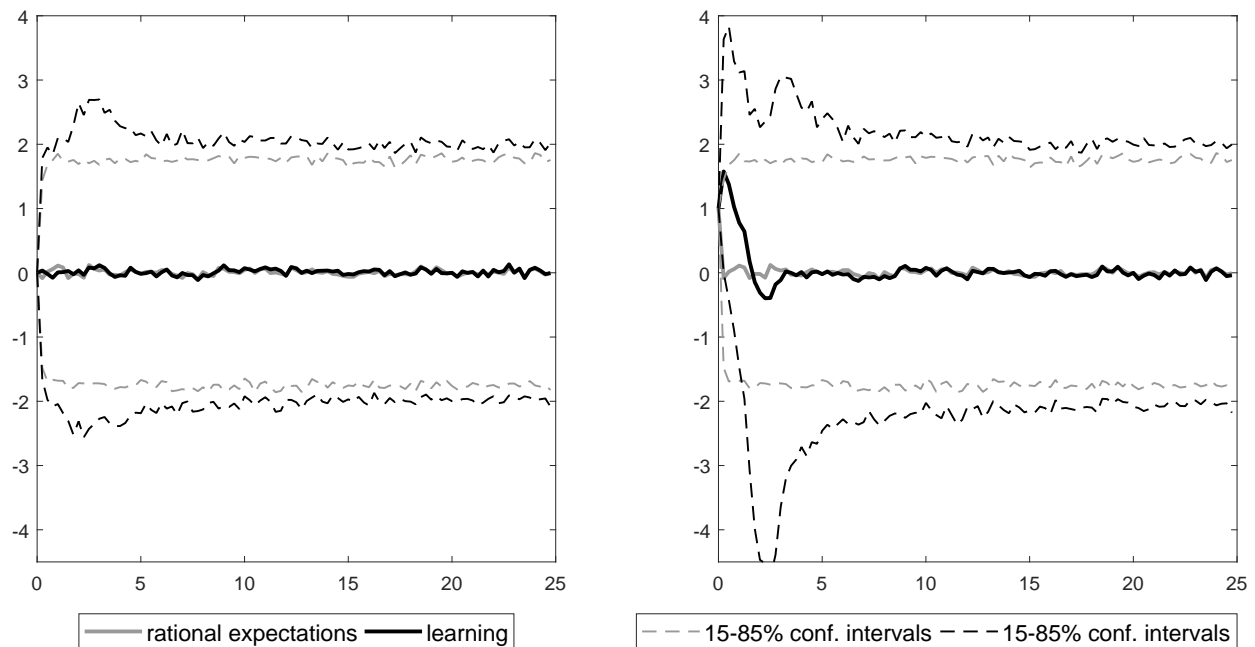


Figure 3.9: Simulation results for inflation over 2000 simulations. Solid lines represent median dynamics; dashed lines are the *15th* and *85th* percentiles. Steady-state inflation prior to joining the union is equal to union-wide inflation in the left panel, and higher by one percentage point in the right panel. The timeline represents years after accession.

the dynamics in figure 3.9 are not driven by a particular draw of shocks, as they represent the median over multiple simulations and similar patterns are prevalent if one investigates individual simulation series.

Perhaps even more interesting from the point of view of the European events are the dynamics of net foreign assets, presented in figure 3.10. They can be also considered an equivalent of cumulative current account balances, as the economy always starts with an initial net foreign asset position of zero.

In figure 3.10 the median dynamics of the current account balances in the left panel - i.e. with no differences in pre-accession steady state inflation rates - are similarly virtually the same for the economy under learning and under rational expectations. Despite this similarity in the median dynamics, the model under learning displays a much larger volatility of net foreign assets, indicating that the trade channel is weaker in that case.<sup>28</sup> This comes from the fact that agents first need to learn about the trade channel, which did not play a role before.

A completely different picture arises in the right panel of figure 3.10, which presents the case of higher pre-union inflation rates. Here the economy under learning experiences a sharp accumulation of foreign debt in the initial two to three years - this period is equivalent to

<sup>28</sup>This result is consistent with the findings of Bonam and Goy (2017).

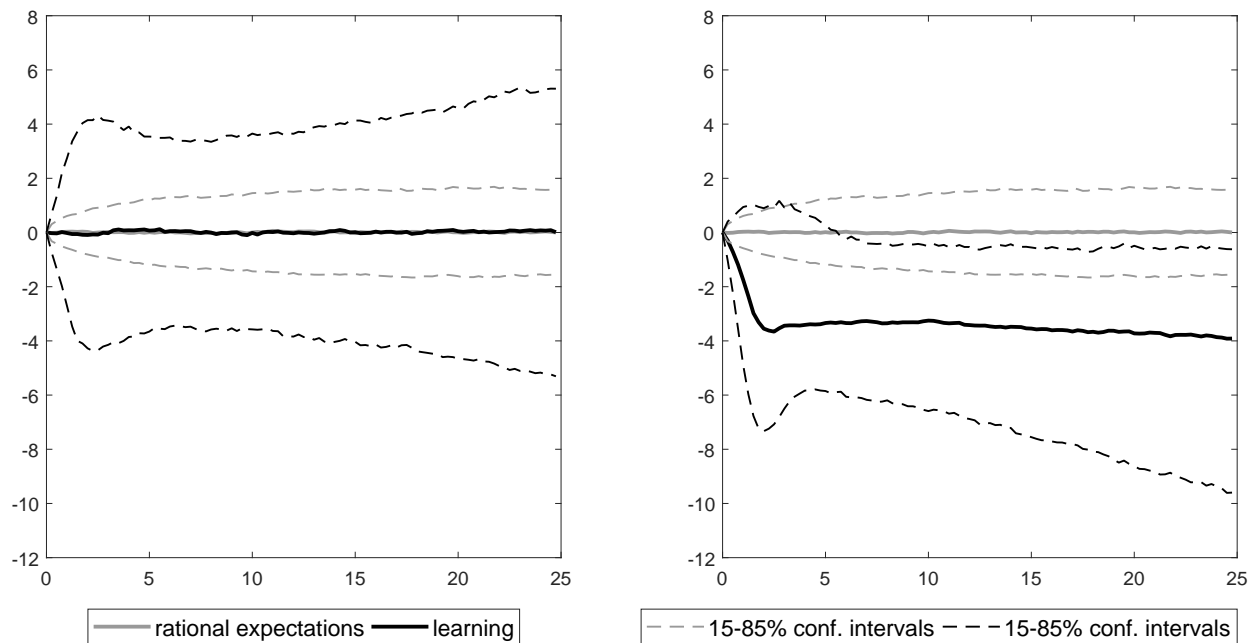


Figure 3.10: Simulation results for net foreign assets (as % of GDP) over 2000 simulations. Solid lines represent median dynamics; dashed lines are the *15th* and *85th* percentiles. Steady-state inflation prior to joining the union is equal to union-wide inflation in the left panel, and higher by one percentage point in the right panel. The timeline represents years after accession.

the time it took the median inflation rate to converge to the target. Interestingly, after the initial period, the net foreign position does not revert towards the zero steady state, but remains relatively constant. A closer investigation of the dynamics reveals that the median current account is slightly negative, implying a slow but continued accumulation of foreign debt. This means that the convergence of inflation rates does not immediately imply the convergence of the remaining variables of the model to its rational expectations equivalents.

In the right panel of figure 3.10 it is also illuminating to consider the behavior of the confidence intervals, or the percentiles of the simulations. The upper confidence interval plateaus quickly at around -1%, whereas the lower interval follows a negative trend from the fifth year after union-accession onwards. This downward trend reveals that a fraction of the simulation series does not stabilize, but keeps on accumulating more foreign debt.

To put this into perspective, the fraction of simulation series with negative net foreign assets is slightly above 90% after ten years of simulations, and above 96% after 25 years. The model does not feature default risk or any debt limits, and therefore is not suited for an analysis of the crisis. But the risk of instability in the model and the existence of the explosive paths of foreign debt for an economy with high inflation experience in the past, resemble well the pre-crisis dynamics in the Eurozone.

To shed further light on the results, one needs to realize that the case for instability within



a monetary union is stronger than in an independent economy. Union-wide monetary policy is not concerned with stabilizing inflation in the small open economy, but rather at the union-wide level. This eliminates the traditionally strongest adjustment mechanism in monetary economics, i.e. the positive reaction of real interest rates to inflation. In fact, union-wide monetary policy actually reverses the mechanism, as real interest rates fall, instead of rising, with the rise of inflation. Then, the only remaining stabilization mechanism relies on trade, but depending on the openness of the economy and the elasticity of substitution between domestic and foreign goods, this channel may be weak. In particular, private agents in the small open economy have not experienced the trade channel prior to joining the monetary union, and thus face a difficult task in learning about it. Moreover, the fall in real interest rates stimulates domestic demand, acting as a counterbalance to the loss of international competitiveness, and potentially overshadowing the trade channel.

## 3.5 Discussion of the results

The research question of this paper has a high degree of policy relevance; therefore, the findings in the previous section have policy implications that are worth exploring. However, before I move to a discussion of the implications of the results it is important to consider their robustness. I concentrate on the sensitivity of the results with respect to the main deviation of this paper from the existing literature on monetary unions, namely the assumption of bounded rationality.

### 3.5.1 Alternative assumptions about learning

The literature on adaptive learning can be criticized on the grounds of providing too much freedom to the researcher who can specify the learning algorithm, as well as the parameters of the learning function.<sup>29</sup> To examine the robustness of my main findings I simulate the model under alternative learning assumptions. First, I stick to the constant gain algorithm, but vary the gain parameter. Figures 3.12-3.14 (in appendix B) present the results for  $g = 0.01, 0.02, 0.05$ . As expected, smaller gain parameters increase the importance of past (i.e. pre-accession) observations, and hence amplify the patterns observed already in figures 3.9 and 3.10. In particular, it takes longer for inflation to converge and the median foreign debt level is larger. The opposite is true for a larger gain parameter; it allows the agents to discard the pre-accession past more quickly, and hence learn faster.

Nevertheless, the quantitative differences are very small and all of the simulations under the alternative gain values generate the same patterns. Under the high inflation history, inflation does not converge immediately and the economy quickly accumulates net foreign debt. The accumulation of debt continues, albeit more slowly, even after inflation converges, and there is a substantial risk of instability. In contrast, none of those dynamics emerge

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<sup>29</sup>This critique is more broadly discussed in Marcet and Nicolini (2003).

under the low inflation history; the median simulation series follows closely the dynamics of the median simulation under rational expectations.

I also consider a popular alternative to the constant gain algorithm, namely the recursive least squares learning algorithm. Under this alternative, the gain parameter  $g$  in equations (3.45) and (3.46) is no longer constant, but instead  $g_t = \frac{1}{t}$ . This approach is equivalent to assuming that the agents run a new ordinary least squares regression each period on the full available sample. The simulation results for the recursive least squares algorithm are presented in figure 3.15 in appendix B. The series are consistent with all the results obtained under the constant gain algorithm, with only minor quantitative differences.

The additional simulation results under alternative learning assumptions allow me to conclude that the mechanism described in the previous section is robust and does not depend on particular modeling choices concerning adaptive learning.

### 3.5.2 Policy implications

The findings in this paper have important policy implications. First of all, they suggest a new candidate for the potential causes of the imbalances within the Eurozone that led to the crisis: differences in expectations, stemming from diverse economic histories across member countries. The importance of past experience for current dynamics implies that it is crucial to monitor country-level expectations within the union. It also calls for a revision of the criteria for an optimal currency area, strengthening the role of economic convergence going further than just synchronization of business cycles.

The findings of the simulation exercise, together with the empirical patterns investigated in section 2, also point to the failure of the convergence criteria embedded in the European Economic and Monetary Union. According to the Maastricht Treaty a country that wants to join the monetary union first needs to satisfy four criteria. Among them is the price stability criterion, which requires the country to achieve a high degree of price stability. The precise definition reads *“that a Member State has a price performance that is sustainable and an average rate of inflation, observed over a period of one year before the examination, that does not exceed by more than 1.5 percentage points that of, at most, the three best performing Member States in terms of price stability.”*

In light of my results, the price stability criterion has at least two major flaws. The compliance with a criterion over just one year might be coincidental, and definitely does not guarantee that inflation expectations are anchored at a given level. More importantly, even if the compliance period were extended, the criterion would still allow the candidate country to record persistently above union-average inflation. A good example are the peripheral EMU countries that experienced above EMU-average inflation over the whole period 1997-1999, and still satisfied the price stability criterion.

An extended version of my model, perhaps estimated on the sample of some EMU countries, could be used to derive an optimal price stability criterion. In principle, the design of such a rule could follow the same mechanism as the design of statistical tests, where one faces a trade-off between type I and type II errors.

Finally, the results of my model show that monetary policy alone is not enough to stabilize the economies of all the member countries. This is a novel finding, as the models typically used in the literature on monetary unions are stable and an appropriate Taylor rule is enough to stabilize the economy. The potential instability of the model under learning creates the space for regional policies. Fiscal or macroprudential policies could react to the build-up of imbalances and steer the economy towards stability.

The effectiveness of fiscal or macroprudential policies in this context poses an interesting question that is left for future research. Another relevant dimension for future research is the role of actual fiscal policies pursued by the EMU member countries in the build-up of the imbalances.

## 3.6 Conclusions

The recent European debt crisis was one of the most severe post-WWII crises and included the rare event of sovereign default in a developed country. Understandably, it triggered a debate about the necessary reform of the monetary union. However, deep reform should be preceded by thorough analysis and understanding of the underlying causes of the crisis. Only well-founded changes to the union can strengthen it and prevent future crises.

In an attempt to contribute to this research agenda, I put forward a potential source of problems within a monetary union: cross-country heterogeneous expectations that are driven by diverse pre-union economic experience. The channel is theoretically novel and has been so far neglected in the discussion of potential causes of the build-up of European imbalances. The plausibility of the importance of expectations is supported by an investigation of EMU pre-crisis empirical patterns. The two main predictors of crisis distress - current account deficits and excessive inflation - correlate well with pre-union inflation and inflation expectations.

I also present a model of a small open economy joining a monetary union, where private agents have imperfect information about the structure of the economy. The main result of this paper is that past inflation experience might have important consequences for a country joining a monetary union, through inflation expectations. In particular, these diverse experiences and expectations may lead to a lack of inflation convergence and an accumulation of large imbalances within the union. The simulations of the model reproduce the main patterns within the Eurozone.

In the model, persistent foreign debt accumulation is possible through a temporary instability. The instability arises when the channels stabilizing the economy change. The traditional monetary policy and nominal exchange rate channels are shut down, since a country joining the union has to transfer its monetary authority to the union-wide central bank and fix its exchange rate. The new stabilizing mechanism is the trade channel: higher prices lead to a loss of competitiveness and, through a fall in demand for domestic goods, to lower inflation. This new channel is weaker than the old channels and the agents need to learn about it. Moreover, the real interest rate channel works now in the opposite direction:

with the interest rate fixed at the union level, real interest rates fall with a rise in inflation.

The simulation results clearly show that the risk of instability arises when the pre-accession steady state inflation rate exceeds the union's inflation target. The convergence of inflation to the new target is then slow and comes with an accumulation of foreign debt. This debt does not return to its steady state level of zero, even after the inflation rate reaches the new target. On the contrary, the accumulation of debt continues, albeit, at a slower pace. The simulation results are robust to changes in the learning assumptions.

The findings of my paper also have important policy implications. The potential of instability arising due to cross-country heterogeneity implies that union-wide monetary policy is not enough to stabilize regional economies. Hence, my paper calls for a more pronounced role of regional policies. The findings also highlight the necessity for closer monitoring of expectations within a monetary union, and a revision of the convergence criteria.

The work could be further extended by estimating the model for the EMU countries. Such an extension would allow to quantify the role of experiences and expectations in the build-up of imbalances within the Eurozone. Another promising research direction is the optimal design of regional fiscal and macroprudential policies in light of the potential instability.

### 3.7 Appendix A - Equations for region F

A typical household in the F region maximizes his lifetime utility, which can be represented by

$$U_t^* = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_{\tau}^*, L_{\tau}^*), \quad (3.47)$$

subject to a sequence of budget constraints of the form

$$P_{\tau}^* C_{\tau}^* + \frac{B_{F,\tau}^*}{1+i_{\tau}} \leq B_{F,\tau-1}^* + W_t^* L_t^* + \Pi_t^* + \frac{\Pi_{B,t}^*}{1-n}. \quad (3.48)$$

for periods  $\tau = t, t+1, \dots$ , where  $P^*$  is a price index for consumption  $C^*$ ,  $W^*$  is the nominal wage earned by foreign households for supplying their labor to foreign firms and  $\Pi^*$  are profits of foreign firms.

Foreign households have only access to foreign bonds, which are issued with the interest rate  $i_t^*$ . They also receive intermediation profits  $\Pi_B^*$ , which are shared equally among all foreign households. Those profits are the counterpart of the intermediation cost paid by domestic households and are equal to

$$\Pi_B^* = \frac{B_F}{P^*(1+i^*)} \left[ 1 - \frac{Q}{\Psi\left(\frac{SB_F}{P}\right)} \right]. \quad (3.49)$$

Furthermore, since the world consists only of the two regions, the net foreign position of the domestic household have to be the equivalent of the net position of the foreign households

$$nB_F + (1-n)B_F^* = 0. \quad (3.50)$$

The first order conditions of the households are two intertemporal optimality conditions - one for each type of bonds - and a standard intratemporal optimality condition

$$U_C(C_t^*, L_t^*) = \beta E_t \left[ U_C(C_{t+1}^*, L_{t+1}^*) (1+i_t^*) \frac{P_t^*}{P_{t+1}^*} \right], \quad (3.51)$$

$$U_L(C_t^*, L_t^*) = -\frac{W_t^*}{P_t^*} U_C(C_t^*, L_t^*). \quad (3.52)$$

The consumption aggregates for the foreign household are

$$C_H^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \quad (3.53)$$

$$C_F^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}. \quad (3.54)$$

The price aggregates for the foreign household are

$$P_H^* = \left[ \frac{1}{n} \int_0^n p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (3.55)$$

$$P_F^* = \left[ \frac{1}{1-n} \int_n^1 p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (3.56)$$

$$P^* = [\nu^* (P_H^*)^{1-\theta} + (1-\nu^*) (P_F^*)^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (3.57)$$

### 3.8 Appendix B - Broader confidence intervals

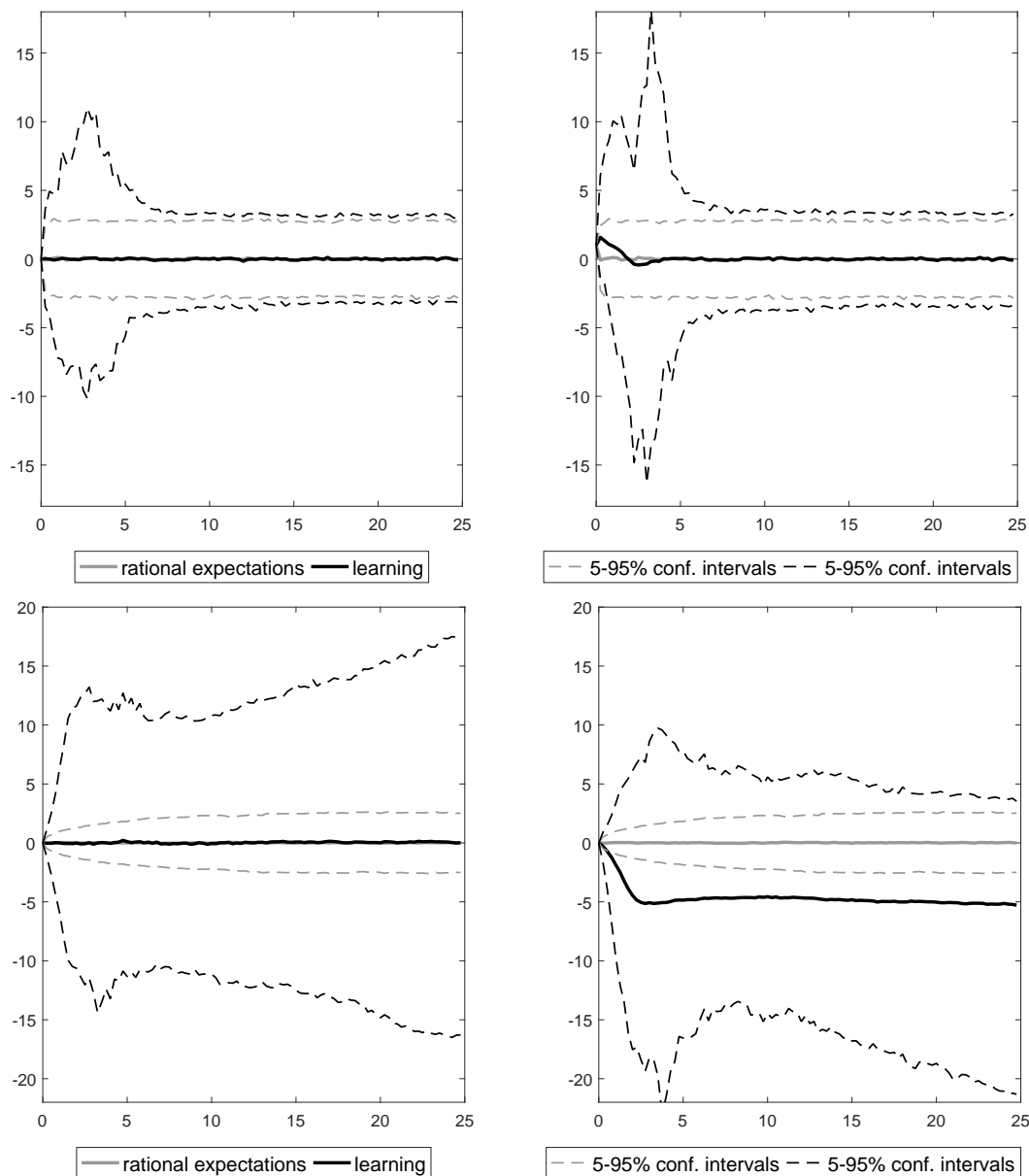


Figure 3.11: Simulation results for inflation (upper panels) and net foreign assets as % of GDP (lower panel) over 2000 simulations. The constant gain parameter is equal to 0.025. Solid lines represent median dynamics, whereas dashed lines are the 5th and 95th percentile. Steady-state inflation prior to joining the union: left panel - equal to union-wide inflation, right panel - higher by one percentage point. The timeline represents years after accession.

### 3.9 Appendix C - Simulations with alternative learning assumptions

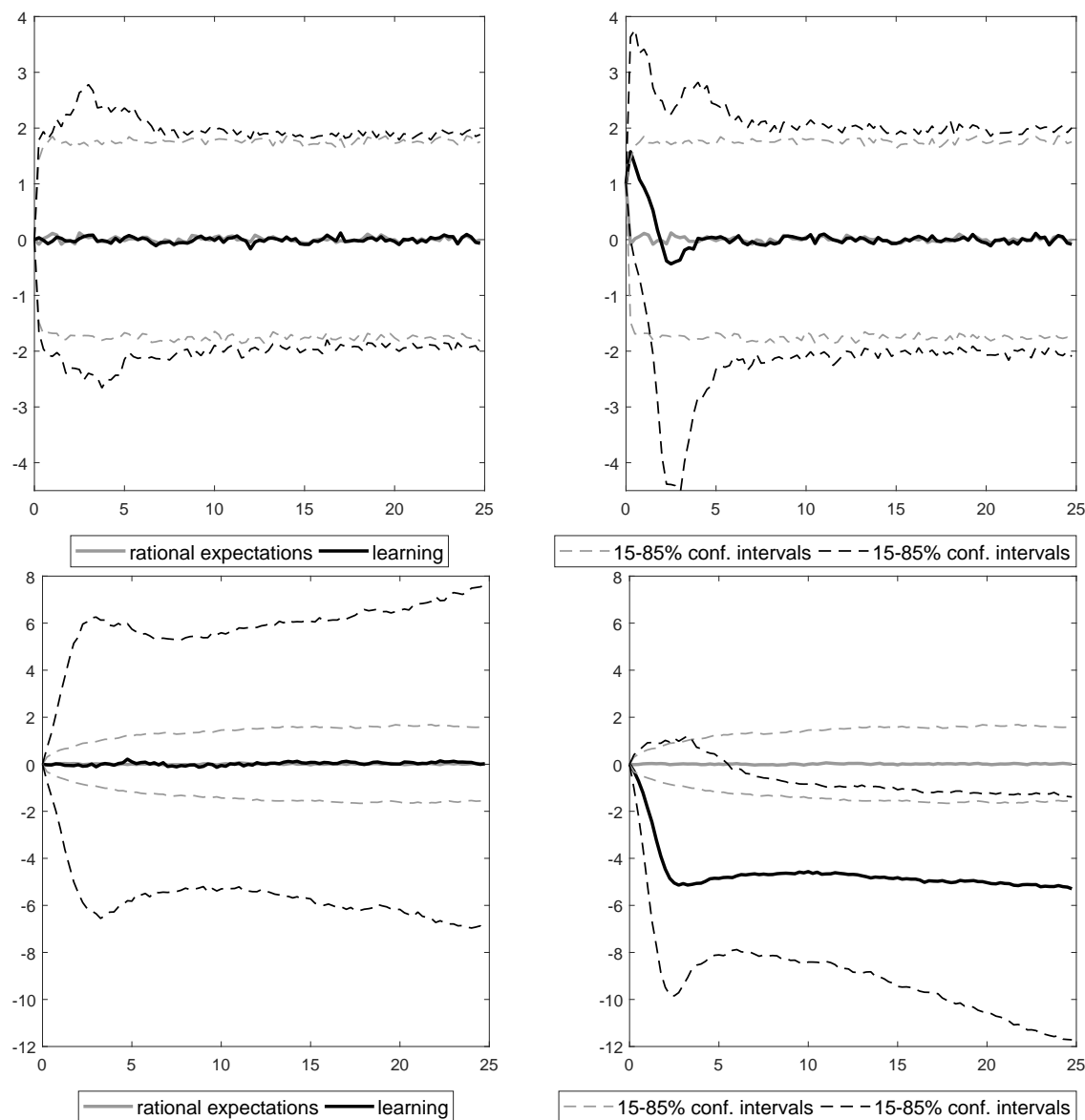


Figure 3.12: Simulation results for inflation (upper panels) and net foreign assets as % of GDP (lower panel) over 2000 simulations. The constant gain parameter is equal to 0.01. Solid lines represent median dynamics, whereas dashed lines are the 15th and 85th percentile. Steady-state inflation prior to joining the union: left panel - equal to union-wide inflation, right panel - higher by one percentage point. The timeline represents years after accession.



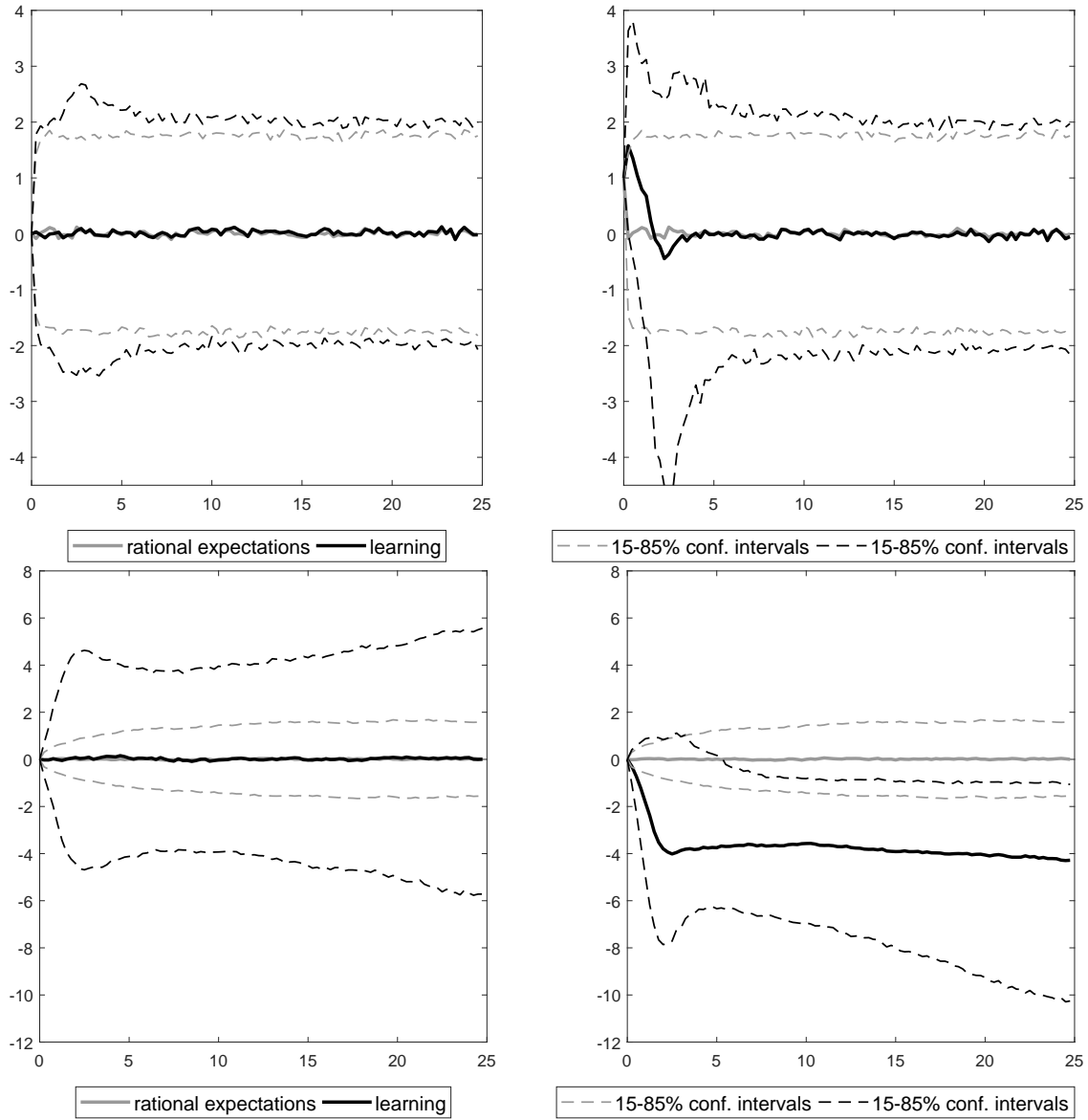


Figure 3.13: Simulation results for inflation (upper panels) and net foreign assets as % of GDP (lower panel) over 2000 simulations. The constant gain parameter is equal to 0.02. Solid lines represent median dynamics, whereas dashed lines are the *15th* and *85th* percentile. Steady-state inflation prior to joining the union: left panel - equal to union-wide inflation, right panel - higher by one percentage point. The timeline represents years after accession.

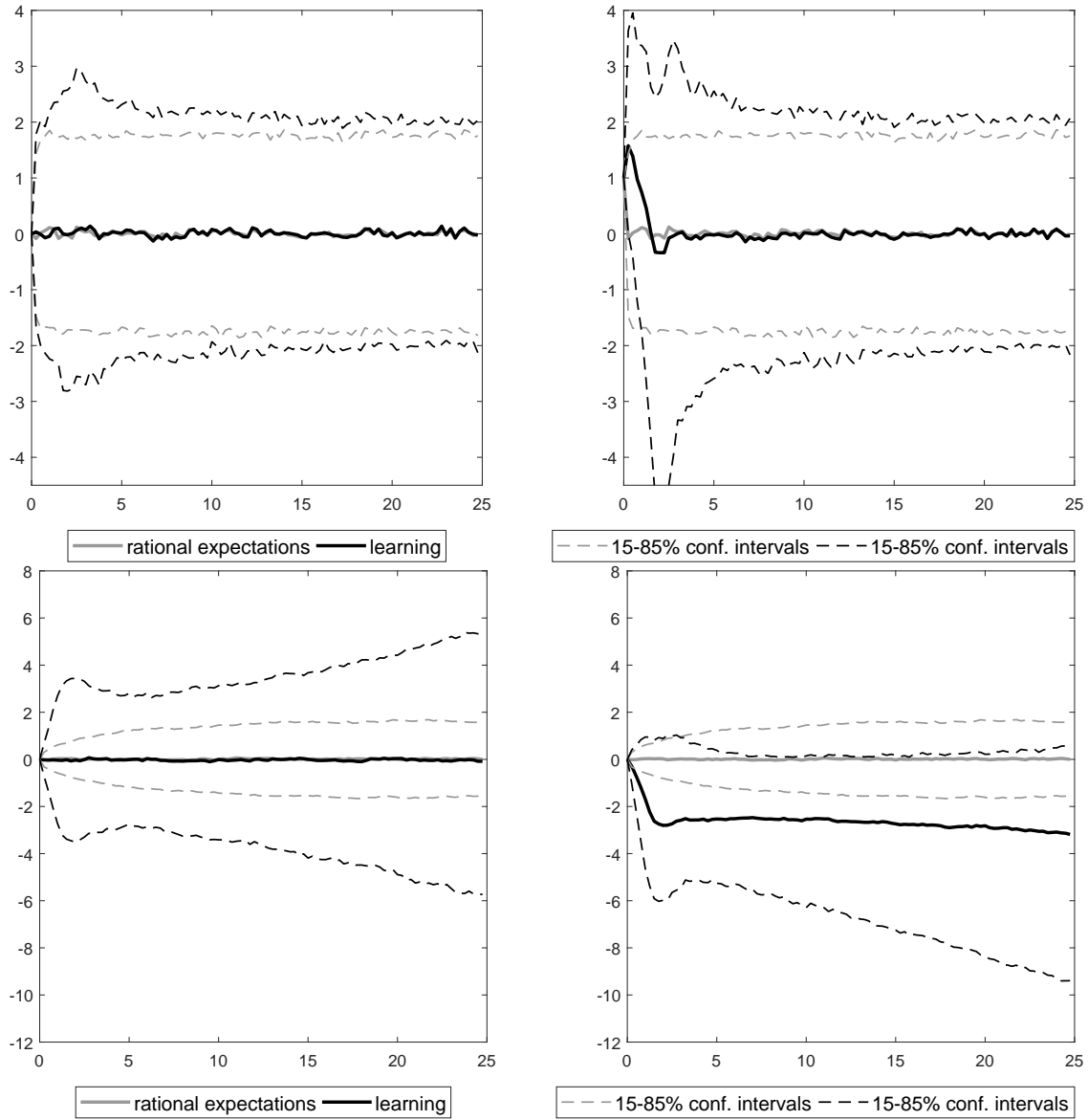


Figure 3.14: Simulation results for inflation (upper panels) and net foreign assets as % of GDP (lower panel) over 2000 simulations. The constant gain parameter is equal to 0.05. Solid lines represent median dynamics, whereas dashed lines are the *15th* and *85th* percentile. Steady-state inflation prior to joining the union: left panel - equal to union-wide inflation, right panel - higher by one percentage point. The timeline represents years after accession.

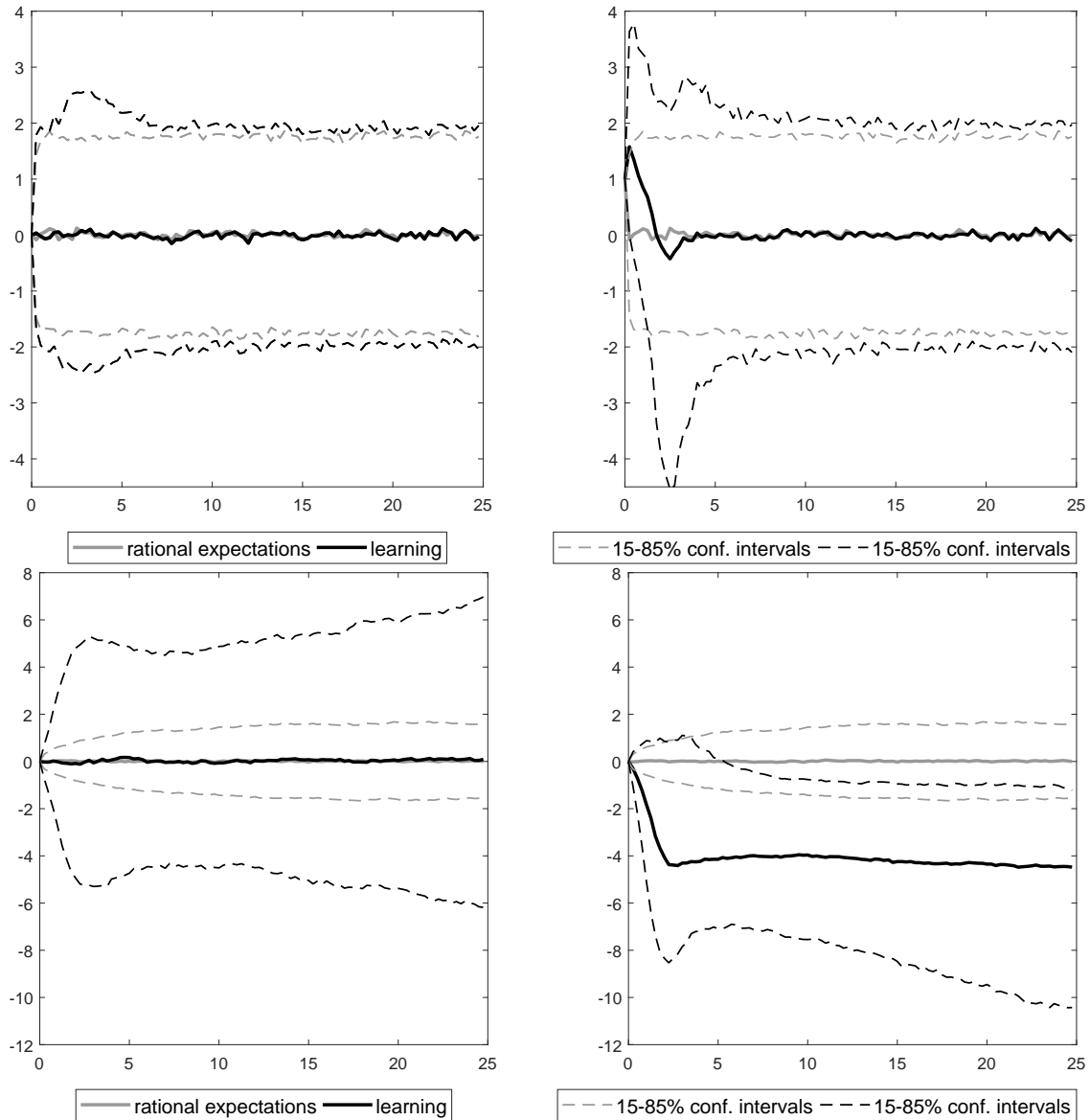


Figure 3.15: Simulation results for inflation (upper panels) and net foreign assets as % of GDP (lower panel) over 2000 simulations. The agents use the recursive least squares learning algorithm. Solid lines represent median dynamics, whereas dashed lines are the *15th* and *85th* percentile. Steady-state inflation prior to joining the union: left panel - equal to union-wide inflation, right panel - higher by one percentage point. The timeline represents years after accession.

# Chapter 4

## Exact Present Solution with Consistent Future Approximation: A Gridless Algorithm to Solve Stochastic Dynamic Models

1

### 4.1 Introduction

*Carpe diem, quam minimum credula postero*  
(seize the present, trust tomorrow e'en as little as you may)

The difficulty in solving stochastic dynamic rational expectations models is that agents are forward looking, which means that one cannot determine this period's behavior unless one knows next period's behavior. It is standard practice to find a recursive solution, which means that the model's solution (as a function of the model's state variables) are the same each period. A model solution is then a set of functions with the following property: If this set of functions is used to describe future model outcomes, then this period's model outcomes are described by the same set of functions. That is, a solution is a fixed point in function space.<sup>2</sup>

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<sup>1</sup>This chapter is based on joint work with Wouter Den Haan and Pontus Rendahl.

<sup>2</sup>The arguments of the functions are the model's state variables.

By contrast, if future model outcomes are known (as a function of next period’s state variables), then the problem is much simpler. If the model has  $n$  endogenous variables, then finding the solution for a particular set of state variables would require solving a system of  $n$  equations in  $n$  unknowns. That is, instead of finding a solution in function space, one only has to find a solution in  $n$ -dimensional Euclidian space. Building on this logic, Den Haan and De Wind (2012) propose to describe future behavior using a simple perturbation approximation and solve for this period’s behavior from the original model equations. Since the original model equations are used, this solution does take into account nonlinearities, uncertainty, and any possible interaction between the two. Unfortunately, describing future behavior with a simple perturbation solution can be so inaccurate that this period’s choices are inaccurate as well.

The algorithm that we propose eliminates the ad hoc choice of describing future behavior, but still finds the solution by solving a simple system of equations. The idea is the following. Given future behavior, one can solve for this period’s behavior. If one can solve for this period’s solution, then one can also solve for the derivatives of this period’s solution. The key step of our algorithm is to use these derivatives to construct a Taylor series expansion, which is then used to describe next period’s behavior. More formally, we solve for model outcomes in each period using a small system of equations that contains the original model equations as well as some additional equations which ensure that next period’s behavior is a Taylor series expansion of this period’s behavior, around this period’s state variables. Since this period’s outcomes are solved using the original model equations, they incorporate any possible nonlinearities, uncertainty, and interactions thereof. We refer to our algorithm as the Exact-Today (ET) algorithm since this period’s outcomes are an exact solution of the model equations and we only approximate next period’s outcomes.

In contrast to perturbation methods, our Taylor series expansion is only used to characterize next period’s behavior in the model equations. Actual behavior is solved from the original model equations and incorporates any possible consequences of nonlinearities and/or uncertainty. The algorithm’s advantage relative to projection methods is that it does not require constructing a grid. In more complex models, it may be difficult to construct a grid such that all calculations make sense at all nodes on the grid.<sup>3</sup> Another problem with grid-based methods is the curse of dimensionality, in which the complexity of the problem increases exponentially with the number of state variables. In contrast, the complexity of our algorithm only increases linearly with the number of state variables.

There are already quite a few algorithms to solve stochastic dynamic models. To document the usefulness of our algorithm, we implement it using a challenging model. This is the model considered in Coeurdacier et al. (2011) (CRW). The model is cast in partial equilibrium in which an agent faced with stochastic income and stochastic returns decides how much to save and how much to consume. This type of model is often used in open economy macroeconomics to describe small open economies. The difficulty of this model lies

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<sup>3</sup>As discussion in section 4.2.3, this matters in practice. For example, particular combinations of state variables may make no sense. The model would never end up at those points, but one may not be aware of this when constructing the grid.

not in its size but in that uncertainty is key in keeping the model well behaved. There exist no steady state and savings diverge absent a sufficient amount of uncertainty. Moreover, savings also diverge if uncertainty is too large.

CRW propose a modified perturbation method to solve this model, which entails taking a second-order approximation of the Euler equation and finding a consistent perturbation solution. We document that their solution is actually very inaccurate. By contrast, the solution generated by our algorithm is shown to be very close to an accurate projection algorithm that solves the problem using more than eight million nodes.<sup>4</sup>

Section 4.2 describes and motivates the algorithm. Section 4.3 describes the model of Coeurdacier et al. (2011), which we use as a test case for our algorithm. Section 4.4 documents the accuracy of our algorithm and the algorithm proposed in Coeurdacier et al. (2011).

## 4.2 The ET algorithm

In this section, we describe the method, which is followed by a short subsection describing what one would actually have to program. The method's merits are discussed in the last subsection.

We focus on a class of models that can be represented by the following system of equations:

$$0 = \mathbb{E}_t [f(x_{t-1}, x_t, x_{t+1}, z_t, z_{t+1})], \quad (4.1)$$

where  $x_t$  contains the endogenous variables,  $z_t$  the exogenous random variables,  $\mathbb{E}_t [\cdot]$  denotes the expectation operator, and  $f(\cdot)$  is a known, typically nonlinear, function.<sup>5</sup> To simplify notation, we focus on the case when both  $x_t$  and  $z_t$  are scalars, but the method easily generalizes to the multidimensional case. For the same reason, we assume that  $z_t$  is a simple AR(1) process. That is,

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad (4.2)$$

$$\mathbb{E}_t [\varepsilon_{t+1}] = 0. \quad (4.3)$$

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<sup>4</sup>Eight million nodes may seem overly prudent. However, the projection method using "only" 441,000 actually performs worse than our proposed algorithm (!), and a very fine grid is therefore necessary in order to adequately assess the accuracy of the various approaches.

<sup>5</sup>When solving stochastic dynamic models, this system of equations is typically the set of first-order and equilibrium conditions. The variables could be transformations of the underlying economic variables such as the logarithm.

### 4.2.1 The method

The objective is to find a recursive solution to equation (4.1) of the following form:

$$x_t = g(x_{t-1}, z_t). \quad (4.4)$$

Projection methods posit an approximating function,  $p(x_{t-1}, z_t; \Phi)$ , where  $\Phi$  is a vector containing the coefficients of the approximating function, and find  $\Phi$  such that equation (4.1) holds exactly, or approximately, on a grid for  $(x_{t-1}, z_t)$ . Perturbation methods also specify a particular functional form and choose the coefficients  $\Phi$  such that the derivatives of  $p(\cdot; \Phi)$  are consistent with the implicit solution of equation (4.1) at the steady state. By contrast, our algorithm, does not restrict the solution of the *current period's* decisions to be of a particular functional form. Instead, it directly solves for  $x_t$  *one point at the time*, by solving a simple system of equations including equation (4.1). By doing so, it preserves the nonlinearities in  $f(\cdot)$  as well as any interaction between nonlinearities in  $f(\cdot)$  and uncertainty about  $\varepsilon_{t+1}$ .

The reader may think that it is not possible to directly solve for  $x_t$  using equation (4.1), because equation (4.1) clearly indicates that to solve for  $x_t$  one needs to know how  $x_{t+1}$  is determined. The underlying principle of ET that makes this possible is that the relationship between  $x_{t+1}$  and *next period's* state variables is imposed to be equal to an approximation of the relationship between  $x_t$  and *this period's* state variables. When solving for  $x_t$ , this approximation for  $x_{t+1}$  is only used to describe the behavior at the relevant points in the state space, namely the points where the economy could find itself next period. Moreover, it is important to realize that the approximation we use to describe next period's behavior is not fixed, but depends on the period- $t$  state variables. This process is repeated each period, so that the actual outcome in period  $t$  is not given by this approximation, but by the solution to the "ET-system" of equations that includes equation (4.1).

The order of approximation for ET may vary depending on the desired degree of accuracy. With first-order ET, however, the value of next period's choice,  $x_{t+1}$ , is approximated by the following linear function:<sup>6</sup>

$$x_{t+1} \approx \tilde{h}(x_t, z_{t+1}; \Phi_t) = \phi_{0,t} + \phi_{x,t}(x_t - \tilde{x}_t) + \phi_{z,t}(z_{t+1} - \tilde{z}_{t+1}), \quad (4.5)$$

where  $\Phi_t$  contains the coefficients of the approximation. That is,<sup>7</sup>

$$\Phi_t = \Phi(x_{t-1}, z_t) \equiv [\tilde{x}_t, \tilde{z}_{t+1}, \phi_{0,t}, \phi_{x,t}, \phi_{z,t+1}]. \quad (4.6)$$

The five elements of  $\Phi_t$  are functions of the state variables,  $x_{t-1}$  and  $z_t$ , and are, thus, time

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<sup>6</sup>Higher-order ET is discussed in appendix D.

<sup>7</sup>It is important to distinguish between  $x_t$  and  $\tilde{x}_t$  and between  $z_{t+1}$  and  $\tilde{z}_{t+1}$ .  $x_t$  and  $z_{t+1}$  are next period's state variables whereas  $\tilde{x}_t$  and  $\tilde{z}_{t+1}$  are coefficients of  $\tilde{h}(\cdot)$ .

varying. For the ET algorithm it does not matter whether  $x_{t-1}$  and  $z_t$  are observations in a simulated time series or an arbitrary point in the state space. Therefore, we focus on the recursive representation of the system of equations, that is,

$$0 = \mathbb{E}[f(x_{-1}, x, x_{+1}, z, z_{+1})], \quad (4.7a)$$

$$z_{+1} = \rho z + \varepsilon_{+1}, \quad (4.7b)$$

$$\mathbb{E}[\varepsilon_{+1}] = 0. \quad (4.7c)$$

Substituting the approximation for  $x_{+1}$  given by equation (4.5) into equation (4.7a) gives

$$\begin{aligned} 0 &= \mathbb{E}\left[f\left(x_{-1}, x, \tilde{h}(x, z_{+1}; \Phi(x_{-1}, z)), z, z_{+1}\right)\right] \\ &= \mathbb{E}\left[f\left(x_{-1}, x, \tilde{h}(x, \rho z + \varepsilon_{+1}; \Phi(x_{-1}, z)), z, \rho z + \varepsilon_{+1}\right)\right] \\ &= \mathbb{E}[F(x_{-1}, x, z, \varepsilon_{+1}; \Phi(x_{-1}, z))]. \end{aligned} \quad (4.8)$$

Since  $\varepsilon_{+1}$  is integrated out, equation (4.8) implicitly defines  $x$  as a function of  $x_{-1}$  and  $z$  for a given  $\Phi(x_{-1}, z)$ . We denote this function by  $h(x_{-1}, z)$ . Thus,

$$0 = \mathbb{E}[F(x_{-1}, h(x_{-1}, z), z, \varepsilon_{+1}; \Phi(x_{-1}, z))]. \quad (4.9)$$

The idea of ET is to choose  $\Phi(x_{-1}, z)$  such that the function that determines  $x_{+1}$ , i.e.,  $\tilde{h}(x, z_{+1}; \Phi(x_{-1}, z))$ , is the Taylor series expansion of the (implicit) function that determines this period's choice,  $h(x_{-1}, z)$ , around this period's state variables. The approximation  $\tilde{h}(x, z_{+1}; \Phi(x_{-1}, z))$  will be accurate if next period's state variables are not that different from this period's state variables. This is a much weaker condition than the one that will ensure accuracy of the perturbation solution, which is that both this period's and next period's state variables are close enough to their steady state values *and* uncertainty is close to zero.

Since  $\tilde{h}(x, z_{+1}; \Phi(x_{-1}, z))$  is a Taylor series expansion of  $h(\cdot)$  around  $x_{-1}$  and  $z$ , it follows that

$$\tilde{x} = x_{-1}, \quad (4.10a)$$

$$\tilde{z}_{+1} = z. \quad (4.10b)$$

That is,  $\tilde{x}$  and  $\tilde{z}_{+1}$  are equal to  $x_{-1}$  and  $z$  respectively. Of course, the values of the coefficients  $\phi_0$ ,  $\phi_x$ , and  $\phi_z$  also depend on  $x_{-1}$  and  $z$ . To understand how these coefficients are



determined, notice that

$$\tilde{h}(\tilde{x}, \tilde{z}; \Phi(x_{-1}, z)) = \phi_0, \quad (4.11a)$$

$$\frac{\partial \tilde{h}(x, z_{+1}; \Phi(x_{-1}, z))}{\partial x} = \phi_x, \quad (4.11b)$$

$$\frac{\partial \tilde{h}(x, z_{+1}; \Phi(x_{-1}, z))}{\partial z_{+1}} = \phi_z. \quad (4.11c)$$

Differentiating equation (4.9) with respect to  $x_{-1}$  and  $z$ , gives

$$0 = \mathbb{E} \left[ \frac{\frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi(x_{-1}, z))}{\partial x_{-1}}}{+ \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi(x_{-1}, z))}{\partial x} \frac{\partial h(x_{-1}, z)}{\partial x_{-1}}} \right], \quad (4.12a)$$

$$0 = \mathbb{E} \left[ \frac{\frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi(x_{-1}, z))}{\partial z}}{+ \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi(x_{-1}, z))}{\partial x} \frac{\partial h(x_{-1}, z)}{\partial z}} \right]. \quad (4.12b)$$

From this equation, we obtain expressions for  $\partial h(x_{-1}, z) / \partial x_{-1}$  and  $\partial h(x_{-1}, z) / \partial z$ . From now on, we suppress the dependence of  $\Phi(x_{-1}, z)$  on  $x_{-1}$  and  $z$ , although this dependence is a key feature of ET. The key approximation step of ET is to assume that the coefficients of the approximation for next period's choices are set as follows:

$$\phi_0 = \tilde{h}(\tilde{x}, \tilde{z}; \Phi) = h(x_{-1}, z), \quad (4.13a)$$

$$\phi_x = \left. \frac{\partial \tilde{h}(x, z_{+1}; \Phi)}{\partial x} \right| = \left. \frac{\partial h(\hat{x}_{-1}, \hat{z})}{\partial \hat{x}_{-1}} \right|_{\substack{\hat{x}_{-1}=x_{-1}, \\ \hat{z}=z}}, \quad (4.13b)$$

$$\phi_z = \left. \frac{\partial \tilde{h}(x, z_{+1}; \Phi)}{\partial z_{+1}} \right| = \left. \frac{\partial h(\hat{x}_{-1}, \hat{z})}{\partial \hat{z}} \right|_{\substack{\hat{x}_{-1}=x_{-1}, \\ \hat{z}=z}}. \quad (4.13c)$$

In words, the function determining *next period's* choice,  $x_{+1} = \tilde{h}(x, z_{+1}; \Phi)$ , is equal to a local approximation of *this period's* choice,  $x = h(x_{-1}, z)$ , around this period's values of the state variables. Equation (4.13a) immediately implies that  $\phi_0 = x$ . This leaves us with the following system of three equations to solve for  $x$ ,  $\phi_x$ , and  $\phi_z$ :<sup>8</sup>

$$0 = \mathbb{E}[F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)], \quad (4.14a)$$

$$0 = \mathbb{E} \left[ \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)}{\partial x_{-1}} + \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)}{\partial x} \phi_x \right], \quad (4.14b)$$

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<sup>8</sup>Although  $\tilde{x} = x_{-1}$  and  $\tilde{z}_{+1} = z_t$ , these variables should be treated as constant coefficients when differentiating  $F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)$ .

$$0 = \mathbb{E} \left[ \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)}{\partial z} + \frac{\partial F(x_{-1}, x, z, \varepsilon_{+1}; \Phi)}{\partial x} \phi_z \right]. \quad (4.14c)$$

Lastly, we have to deal with the expectations operator. If  $\varepsilon_{+1}$  has discrete support, then the expectations operator can be replaced by a sum of the possible outcomes,  $\varepsilon_j$ , and the associated weights,  $\pi_j$ , with  $j \in \{1, \dots, J\}$ . If  $\varepsilon_{+1}$  has continuous support, then one can use a numerical integration procedure, which boils down to doing the same. For example, if  $\varepsilon_{+1}$  has a Normal distribution with mean zero and standard deviation equal to  $\sigma$ , then using Gauss-Hermite quadrature implies that

$$\varepsilon_j = \sqrt{2}\sigma\zeta_j, \text{ and} \quad (4.15)$$

$$p_j = \frac{\omega_j}{\sqrt{\pi}}, \quad (4.16)$$

where the  $\zeta_j$ 's and the  $\omega_j$ 's are the Gauss-Hermite nodes and weights, respectively.<sup>9</sup>

**Relationship to Den Haan and De Wind (2012)** Den Haan and De Wind (2012) propose to set  $\tilde{h}(x, z_{+1}; \Phi)$  equal to a standard perturbation solution. Equation (4.8) can then also be used to solve for  $x$  for given values of  $x_{-1}$  and  $z$ . In this approach, the value of  $\Phi$  is time invariant and independent of  $x_{-1}$  and  $z$ . The solution for  $x$  would still take into account the nonlinearity of  $f(\cdot)$  and the uncertainty about  $\varepsilon_{+1}$ . However, the standard perturbation solution may not be a reliable representation of the solution for  $x_{+1}$ , because it is a local approximation around the steady state values of the state variables when there is no uncertainty. By contrast, ET sets the choice for  $x_{+1}$  equal to the behavior according to a local approximation around this period's state variables and uncertainty is not reduced to zero.<sup>10</sup>

## 4.2.2 What actually needs to be programmed

The key step is to construct a function that defines the three error terms associated with equation (4.14) as a function of the arguments,  $x$ ,  $\phi_x$ , and  $\phi_z$  for given values of  $x_{-1}$  and  $z$ .

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<sup>9</sup>Intuitively, one can think of the  $\sqrt{2}\sigma\zeta_j$  terms as the possibly outcomes of  $\varepsilon_{+1}$  and of the  $\omega_j/\sqrt{\pi}$  terms as the associated probabilities. Simple subroutines exist to generate the Gauss-Hermite quadrature nodes and weights. Numerical integration techniques are very powerful. For example, if  $\varepsilon_{+1}$  is distributed according to a Normal distribution, and one wants to calculate  $\mathbb{E}[F(\varepsilon_{+1})]$ , then the Gauss-Hermite quadrature approximation with  $J$  nodes gives the exact for *all*  $(2J-1)^{\text{th}}$ -order polynomials, and an accurate answer when  $F(\cdot)$  is well approximated by a  $(2J-1)^{\text{th}}$  order polynomial.

<sup>10</sup>The procedure used in Den Haan and De Wind (2012) improves substantially upon standard perturbation solutions for some models. However, it is shown to be only slightly better than the regular perturbation solution for others.

Those error terms are given by

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^J F(x_{-1}, x, z, \varepsilon_j; \Phi) p_j \\ \sum_{j=1}^J \left( \frac{\partial F(x_{-1}, x, z, \varepsilon_j; \Phi)}{\partial x_{-1}} + \frac{\partial F(x_{-1}, x, z, \varepsilon_j; \Phi)}{\partial x} \phi_x \right) p_j \\ \sum_{j=1}^J \left( \frac{\partial F(x_{-1}, x, z, \varepsilon_j; \Phi)}{\partial z} + \frac{\partial F(x_{-1}, x, z, \varepsilon_j; \Phi)}{\partial x} \phi_z \right) p_j \end{bmatrix}, \quad (4.17)$$

whereas before

$$F(x_{-1}, x, z, \varepsilon_j; \Phi) = f(x_{-1}, x, \tilde{h}(x, \rho z + \varepsilon_j; \Phi), z, \rho z + \varepsilon_j), \quad (4.18)$$

$$\tilde{h}(x, \rho z + \varepsilon_j; \Phi) = \phi_0 + \phi_x(x - \tilde{x}) + \phi_z(z_{+1} - \tilde{z}), \quad (4.19)$$

and

$$\Phi = [\tilde{x}, \tilde{z}, \phi_0, \phi_x, \phi_z] = [x_{-1}, z, x, \phi_x, \phi_z]. \quad (4.20)$$

Defining these error terms requires taking derivatives. As shown in appendices D and F, this is a very simple programming step if one has access to a symbolic toolbox even if one considers higher-order approximations. For quite a few problems this could also be done by hand.

The next and final step is to use an equation solver to find the values of  $x$ ,  $\phi_x$ , and  $\phi_z$  that set the three error terms equal to zero for given values of  $x_{-1}$  and  $z$ .

### 4.2.3 Merits of the ET algorithm

The ET algorithm shares with projection methods the property that the numerical solution incorporates the uncertainty and nonlinearity underlying the original system of equations without modifying  $f(\cdot)$  and with either no modification of the expectations operator (in case  $\varepsilon_{+1}$  has discrete support) or with a minor modification (in case  $\varepsilon_t$  has continuous support). This is even true for first-order ET, since the period- $t$  choice variables are always solved from the original nonlinear set of first-order conditions. The approximation aspect of the ET solution *only* affects the characterization of *next period's* policy functions. Moreover, the derivatives of the Taylor-series expansion used to characterize next period's behavior vary with the value of this period's state variables. By contrast, the traditional perturbation method characterizes this period's behavior with a particular *fixed* Taylor series expansion using the state variables *and* the amount of uncertainty as its arguments. The derivatives used in this Taylor-series approximation correspond to the derivatives at the steady state. This means that the accuracy of the standard perturbation solution is only guaranteed if the state variables are close to their steady state values and the model's volatility parameters are close to zero. Higher-order perturbation does incorporate the effect of uncertainty, but imposes that the effect of uncertainty on model outcomes is equal to an extrapolation of the effect of uncertainty on behavior at a situation when there is no uncertainty to begin with.

Projection methods, on the other hand, solve a much larger set of simultaneous equations, namely the model equations at *all* grid points.<sup>11</sup> By contrast, ET can solve for the choices at a particular point in the state space without having to solve for the solution at other points in the state space. For the application considered in this paper, which is quite challenging, the projections algorithm has to solve a simultaneous system of more than eight million nodes, whereas second-order ET needs to solve a system of ten equations and ten unknowns.<sup>12</sup> ET can do so because it focuses on behavior at the *relevant* points in the state space, namely those where the economy could be in the next period, and it does so by imposing that next period's choices are determined by a Taylor-series expansion of this period's choices. It is true that ET has to solve this system many times, but it is a fixed system of equations, so it only has to be defined once.

It is important to realize that the comparison of standard projection methods and ET is not only an issue of comparing the speed of solving a large system of equations once versus solving a smaller system of equations many times. There are two potential advantages of ET. The first is that ET is not subject to the curse of dimensionality. Whereas the number of elements of a regular grid increases exponentially with the number of state variables, the number of unknowns the ET algorithm has to solve for increases at a slower rate. In particular, it increases at a linear rate if a first-order approximation is used to describe next period's behavior.<sup>13</sup> A second advantage of ET is related to the points considered in calculating the numerical approximation. To apply standard projection methods, one has to construct a grid and the algorithm must be able to find a sensible solution at *all* grid points. But there may be points in the state space where some calculations are not well defined.<sup>14</sup> This is not a problem if the economy never reaches those points. Unfortunately, when constructing the grid one typically would not know which points in the state space one has to exclude for this reason. This is a relevant problem in practice and has motivated some researchers to develop methods that – like ET – use simulated time paths instead of pre-specified grids.<sup>15</sup>

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<sup>11</sup>If the number of nodes exceeds the number of coefficients, then a minimization routine would be used.

<sup>12</sup>First-order ET requires solving a system of three equations in three unknowns.

<sup>13</sup>Recall that even the first-order ET solution takes into account the full nonlinearity and uncertainty of the underlying system.

<sup>14</sup>One possibility is that there is simply no solution at these nodes. Another possibility is that there is no solution for the particular functional form chosen for the approximation. But even if neither is the case, then it still may be the case that some points in the state space are particularly problematic *in the process of* finding the numerical solution, which typically involves considering many values for  $\Phi$ .

<sup>15</sup>Simulation-projection methods are not new. An example is the Parameterized Expectations Algorithm of Den Haan and Marcet (1990). As pointed out in Judd (1992) and documented in Den Haan (1995), those earlier simulation approaches can be quite inefficient in that accuracy of the projection's part of the algorithm, i.e., the regression phase, requires a very large number of observations. Judd et al. (2011) develop a simulation-based method with a more efficient projection's element.

### 4.3 The model of Coeurdacier, Rey, and Winant (CRW)

We use the algorithm to solve the model considered in Coeurdacier et al. (2011). This is a challenging model to solve, since the model has no steady state and is not well defined when there is no or not a sufficient amount of uncertainty.<sup>16</sup> Moreover, as discussed below, the model can also generate non-stationary behavior if there is too much uncertainty and there are some important nonlinearities.

In the CRW model, an infinitely-lived agent decides how much to consume,  $c_t$ , and how much funds to invest and carry over to the next period,  $w_t$ . Both the agent's income,  $y_t$ , and the rate of return,  $r_t$ , are exogenous stochastic processes. The solution to the agent's problem is characterized by the following set of equations:

$$c_t^{-\gamma} = \beta \mathbb{E}_t [c_{t+1}^{-\gamma} r_{t+1}], \quad (4.21)$$

$$c_t + w_t = y_t + w_{t-1} r_t, \quad (4.22)$$

$$y_t = (1 - \rho_y) \bar{y} + \rho_y y_{t-1} + \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, \sigma_y^2), \quad (4.23)$$

$$r_t = (1 - \rho_r) \bar{r} + \rho_r r_{t-1} + \varepsilon_{r,t}, \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2). \quad (4.24)$$

If there would be no uncertainty, the Euler equation reduces to

$$c_{t+1} = (\beta \bar{r})^{1/\gamma} c_t, \quad (4.25)$$

which implies that consumption would either increase without bound or go to zero unless  $\beta = 1/\bar{r}$ . That is, there is no steady-state solution when  $\sigma_y = \sigma_r = 0$ .<sup>17</sup> In the CRW model, there are no constraints on  $w_t$  and short positions of any size are allowed. Thus, uncertainty is key for the ability of the model to generate a stationary solution.<sup>18</sup>

#### 4.3.1 Natural borrowing constraint.

For the projections algorithm to work smoothly, we found it necessary to impose the natural borrowing constraint. That is, the maximum amount that an agent can borrow is such that their income will always be enough to cover interest payments, even in the worst possible

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<sup>16</sup>Standard projection methods can be used to solve this model, but it is not a trivial problem for these methods either. Since uncertainty is essential, one cannot follow standard practice of first solving a model with little uncertainty and then gradually moving to the case with the desired levels of uncertainty.

<sup>17</sup>When  $\beta = 1/\bar{r}$ , then there would be a steady state with  $c_t = \bar{y} + (r-1)w_0$ , but any amount of uncertainty would cause the solution to be non-stationary.

<sup>18</sup>In appendix A, we provide some intuition for this statement. A more formal analysis of the model's properties can be found in Chamberlain and Wilson (2000).

circumstances. In particular, when solving for the projections solution, we impose that

$$w \geq -\frac{y_{\text{low}}}{r_{\text{high}} - 1}, \quad (4.26)$$

where  $y_{\text{low}}$  is the lowest possible value for  $y_t$  and  $r_{\text{high}}$  the highest possible value for  $r_t$ .<sup>19</sup> For the parameters considered, this constraint turns out to be never binding, even in the very long simulations considered. As discussed below, this does not imply that it does not affect the behavior of the agent's choices. Neither the CRW algorithm nor the ET algorithm impose the natural borrowing constraint. Nevertheless, as discussed below, the ET solution is characterized by nonlinear behavior that resembles the nonlinearity induced by the natural borrowing constraint.

Table 4.1: Parameter Values

$\sigma_r$	0.00125	$\rho_r$	0.9	$\bar{r}$	1.04152878685	$\gamma$	4
$\sigma_y$	0.01	$\rho_y$	0.9	$\bar{y}$	1		

### 4.3.2 Parameter values.

The parameter values are reported in table 4.1. The two key parameters are the standard deviations of the innovations to output and the rate of return. As documented below, the model variables are quite volatile at the chosen values. Nevertheless, the values for  $\sigma_r$  and  $\sigma_y$  used here are lower than those chosen in Coeurdacier et al. (2011). We do not use the original CRW values for our comparison, because our accurate projection algorithm indicates that the model solution is not well behaved at those values. As documented in appendix E, it displays "escape dynamics". That is, the simulated time paths for wealth will occasionally – but for sustained time periods – diverge to *extremely* high values.<sup>20</sup> The CRW method imposes stationarity even when it does not make sense to do so. Although less restrictive, the ET algorithm also does not seem capable of capturing such explosive behavior.<sup>21</sup> Therefore,

<sup>19</sup>We set  $y_{\text{low}}$  equal to  $\bar{y}$  minus four standard deviations of the unconditional standard deviation of  $y_t$  and we set  $r_{\text{high}}$  equal to  $\bar{r}$  plus four standard deviations of the unconditional standard deviation of  $r_t$ .

<sup>20</sup>Our projection method imposes an upper bound on wealth. Without imposing such an upper bound, the solution diverges to infinity. Outside the (finite) grid, the solution cannot be relied on to be accurate. Our experience indicates that the simulated wealth series reach this upper bound *even* when it is set at extremely high values for a sufficiently long sample. The stochastic innovations for  $r_t$  and  $y_t$  have a normal distribution and could in principle take on extremely high values. This is not the cause of the divergence, since the same explosive behavior is observed when the values of  $r_t$  and  $y_t$  are restricted to be in a bounded set.

<sup>21</sup>With the second-order version, the method fails to find a solution at some point during the simulation. It might still be possible that a higher-order version of the algorithm would be successful.

we chose more moderate volatility levels at which the generated series do not display such divergent behavior. The case with the original CRW parameter values is discussed in more detail in appendix E. We follow CRW and choose the value of  $\bar{r}$  such that the risky steady state value for wealth is equal to 0.<sup>22</sup>

## 4.4 Evaluation of solution methods

In this section, we compare the solutions obtained with the CRW and ET algorithms with an accurate projections-method solution.<sup>23</sup> The CRW procedure is identical to the one used in Coeurdacier et al. (2011). The procedure consist of deriving a second-order approximation of the Euler equation and a perturbation solution that are consistent with each other.<sup>24</sup> To implement the ET procedure, we use a second-order approximation to describe next period's choices.<sup>25</sup> The reason we do not use a first-order approximation is that the ET system of equations does not have a solution at some points in the sample when only a first-order approximation is used.

### 4.4.1 Policy functions

Figure 4.1 plots the policy functions for the change in wealth,  $w - w_{-1}$ , as a function of beginning of period wealth,  $w_{-1}$ , for different values of the realized rate of return and one particular value of income.<sup>26</sup> The top panel displays the results for both the ET and the accurate projection solution. The solutions of these two algorithms are very close for most values of  $w_{-1}$ . The only noticeable differences occur at values of beginning-of-period wealth close the natural borrowing constraint for higher values of the realized rate of return. The projection solution indicates that  $w - w_{-1}$  gets less negative rapidly as  $w_{-1}$  is close to the natural borrowing constraint. The ET solution only uses local information and it does not impose the natural borrowing constraint at all. It is, therefore, remarkable that the ET solution displays a similar nonlinearity close to the natural borrowing constraint.<sup>27</sup>

The figure documents another – more important – nonlinearity: The slope of the policy function for  $w - w_{-1}$  as a function of  $w_{-1}$  depends on the level of this period's realized rate of return,  $r$ . The slope is positive at high rates of return and negative at low rates of return. The level of  $r$  does not only affect this period's resources through  $rw_{-1}$ , it also affects the expected rate of return. Another way to think of the nonlinearity associated with  $r$  is the

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<sup>22</sup>Although, the original system of equations does not have a steady state, the second-order approximation used by CRW does have a steady state, which is referred to as the risky steady state.

<sup>23</sup>The accuracy of the projections-method solution is documented in appendix B.

<sup>24</sup>A description of the CRW procedure is given in appendix C.

<sup>25</sup>Details of the implementation of the ET algorithm are given in appendix D.

<sup>26</sup>The results are very similar for other income values.

<sup>27</sup>Although ET does not impose the natural borrowing constraint, it does avoid choosing wealth levels which could imply very low consumption levels next period, which becomes more likely at very low wealth levels.

following: If  $r$  increases then  $w - w_{-1}$  increases for higher levels of wealth and decreases for lower levels of wealth.<sup>28</sup>

The bottom panel displays the results for the CRW solution. Although the CRW solution is quite accurate for the middle value of  $r$  it predicts that  $\partial w / \partial w_{-1}$  does not depend on  $r$ . As discussed above, even the sign of  $\partial w / \partial w_{-1}$  depends on the level of  $r$  according to the accurate projection solution (and the ET solution).

Figure 4.2 repeats the exercise for the consumption policy function. The ET policy function accurately captures that the slope of the consumption function depends on the level of  $r$ . The CRW policy function does not. Moreover, it overpredicts the consumption choice at high rates of return and underpredicts the consumption choice at low rates of return, especially at lower wealth levels. The nonlinearity associated with the natural borrowing constraint is less pronounced in the consumption policy than in the wealth policy function.

#### 4.4.2 Simulated time paths

Next we compare simulated time paths. These will reveal possible differences and similarities in exactly those parts of the state space that matter for model properties. The model variables are quite persistent, so we use a long simulation of 80,000 observations. Panels A and B of figure 4.3 plot the generated values for wealth and consumption, respectively. The figure shows that the ET solutions for consumption and wealth follow the accurate projection solutions closely. That is clearly not the case for the CRW solution. These figures also document that the series are quite volatile even though the chosen standard deviations are less than those used in Coeurdacier et al. (2011).<sup>29</sup>

Figures 4.4 and 4.5 display wealth and consumption time paths for four shorter samples taken from the full sample. These figures show in greater detail that the behavior of the CRW solution can be quite different from the accurate projection solution. For example in the top right panel of figure 4.4, the wealth series of all three algorithms display a similar downward trend in the beginning of the sample. While this downward trend continues according to the projection and ET solutions, the CRW solution displays a remarkable recovery. In the top left panel, the projection and ET solutions display a sharp increase which is not present in the perturbation solution.

#### 4.4.3 Comovement of CRW & ET time paths with accurate projection solution.

Table 4.2 reports how close the ET and CRW solutions are to the accurate projection solution using average and maximum absolute deviations as well as correlation coefficients. Whereas

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<sup>28</sup>If  $r$  increases, then a saver will have more funds in the current period and a borrower will have less resources. The realization of  $r$  also affects the expected return. The associated income effect also differs for savers and lenders.

<sup>29</sup>To put the volatility in perspective, note that the mean income level is equal to 1.



the correlation between the time series generated with the ET algorithm and those generated by the accurate projection solution are virtually equal to 1, they are considerably lower for the CRW series. For example, the correlation between the CRW wealth series and the

Table 4.2: Distance of ET and CRW Outcomes Relative to Accurate Projection Solution

	ET	CRW
correlation of $w_t$ with projection solution	1.000	0.729
correlation of $\Delta w_t$ with projection solution	1.000	0.798
correlation of HP-filtered $w_t$ with projection solution	1.000	0.790
mean abs. deviation of $w_t$ from projection as fraction of $SD(w_t)$	0.83%	52.3%
max. abs. deviation of $w_t$ from projection as fraction of $SD(w_t)$	3.62%	268.2%
correlation of $c_t$ with projection solution	1.000	0.730
correlation of $\Delta c_t$ with projection solution	1.000	0.891
correlation of HP-filtered $c_t$ with projection solution	1.000	0.885
mean abs. deviation of $c_t$ from projection as fraction of $\bar{y}$	0.21%	13.0%
max abs. deviation of $c_t$ from projection as fraction of $\bar{y}$	0.92%	68.4%

Notes: This table reports different measures to indicate the similarity between data generated by the CRW or the ET algorithm and data generated by an accurate projection algorithm. The statistics are based on a simulation of 80,000 observations.  $SD(w_t)$  stands for the standard deviation of wealth.

projection wealth series is only 0.729. Despite the almost perfect correlation, the ET data are not exactly equal to their projection equivalent. In particular, there is an average absolute difference of 0.83% for wealth and 0.21% for consumption. These compare very favorable to the outcomes for CRW, which are 52.3% and 13%. The results are similar for the maximum differences from the accurate projection solution. For the ET data, the maximum differences are 3.62% for wealth and 0.92% for consumption, whereas the corresponding numbers are 268.2% and 68.4% for the CRW solution.

#### 4.4.4 Comparison of generated moments.

Tables 4.3 and 4.4 report moments according to the three different solutions for the generated wealth and consumption data, respectively. The statistics based on the projection and the ET algorithm are very close to each other. This is true for first and second moments, as well as correlations, of both consumption and wealth. On the other hand, the statistics for the simulations based on the CRW algorithm look significantly different for the simulations based on the CRW algorithm. In particular, the data generated by the CRW algorithm are substantially less volatile; the standard deviation of wealth is only 53% of the projection

Table 4.3: Model Properties According to the Three Algorithms: Wealth

	Projection	ET	CRW
mean of $w_t$	0.934	0.983	0.198
standard deviation of $w_t$	5.99	6.02	3.18
standard deviation of $\Delta w_t$	0.022	0.022	0.017
standard deviation of HP-filtered $w_t$	0.041	0.041	0.031
correlation of $w_t$ and $y_t$	0.033	0.032	0.076
correlation of $w_t$ and $r_t$	-0.004	-0.004	-0.004
correlation of $w_t$ and $c_t$	0.999	0.999	0.998
correlation of $\Delta w_t$ and $\Delta y_t$	0.168	0.168	0.216
correlation of $\Delta w_t$ and $\Delta r_t$	0.068	0.069	0.060
correlation of $\Delta w_t$ and $\Delta c_t$	0.375	0.376	0.327
correlation of HP-filtered $w_t$ and $y_t$	0.118	0.117	0.144
correlation of HP-filtered $w_t$ and $r_t$	0.048	0.049	0.045
correlation of HP-filtered $w_t$ and $c_t$	0.432	0.433	0.370

Notes: This table reports model properties for data generated by the indicated algorithm. The results are based on a simulation of 80,000 observations.

analogue and the standard deviation of consumption is only 54% of the projection analogue. Moreover, the precautionary saving motive is substantially weaker with CRW, as the mean wealth level based on the projection and ET algorithms are close to a one period average income, whereas for CRW it is below 20% of the average period income.<sup>30</sup>

## 4.5 Conclusions

This paper proposes an algorithm for solving stochastic dynamic models. The algorithm tackles the difficulty embedded in modern economic models in which the system of equations of the model involves not only current (and past) variables but also the current expectations of future variables. We propose reducing the difficulty of the problem by approximating future variables with a Taylor series around current variables. To be able to identify the additional parameters included in the Taylor approximations, we extend the original system of equations with derivatives of the original equations.

<sup>30</sup>Nevertheless, it is important to highlight that the CRW algorithm generates correlations similar to the accurate projection solution (but not as similar as ET). This strength of the CRW algorithm might be coming from the use of covariances and variances in obtaining the solution.

Table 4.4: Model Properties According to the Three Algorithms: Consumption

	Projection	ET	CRW
mean of $c_t$	1.039	1.041	1.008
standard deviation of $c_t$	0.250	0.251	0.134
standard deviation of $\Delta c_t$	0.0038	0.0038	0.0036
standard deviation of HP-filtered $c_t$	0.0050	0.0050	0.0046
correlation of $c_t$ and $y_t$	0.057	0.056	0.121
correlation of $c_t$ and $r_t$	-0.020	-0.020	-0.037
correlation of $\Delta c_t$ and $\Delta y_t$	0.747	0.747	0.808
correlation of $\Delta c_t$ and $\Delta r_t$	-0.455	-0.452	-0.560
correlation of HP-filtered $c_t$ and $y_t$	0.723	0.723	0.788
correlation of HP-filtered $c_t$ and $r_t$	-0.441	-0.439	-0.546

Notes: This table reports model properties for data generated by the indicated algorithm. The results are based on a simulation of 80,000 observations.

Despite the additional equations, our algorithm does not suffer under the curse of dimensionality, as it is a local method, and thus does not require a grid. The approximation can be applied locally at any chosen point of the state space. At the same time, the proposed solution method preserves the non-linearities and uncertainty of the original model, which is an advantage over the standard local approximation method, i.e. perturbation.

We apply our algorithm to a simple but challenging model of a small open economy, which does not have a deterministic steady state and requires uncertainty to be stable. We show that our algorithm can generate accurate solutions even when the model series are quite volatile.

We also test the algorithm proposed in Coeurdacier et al. (2011) and conclude that it is not accurate. Even more importantly, it can generate misleading stable dynamics when the model is unstable, as presented in Appendix E.

## 4.6 Appendix A - The role of uncertainty for stationarity

The following second-order approximation of the Euler equation provides some intuition for the result that the solutions for  $c_t$  and  $w_t$  could be stationary because of uncertainty.

$$\begin{aligned}
c_t^{-\gamma} &\approx \beta \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma} \\
&\quad + \\
&\quad \gamma(\gamma+1) \beta \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma-2} \mathbb{V}_t(c_{t+1}) \\
&\quad - \beta \gamma \mathbb{E}_t[c_{t+1}]^{-\gamma-1} \mathbb{C}_t(c_{t+1}, r_{t+1}),
\end{aligned} \tag{4.27}$$

where

$$\mathbb{V}_t(c_{t+1}) \equiv \mathbb{E}_t[(c_{t+1} - \mathbb{E}_t[c_{t+1}])^2], \tag{4.28}$$

$$\mathbb{C}_t(c_{t+1}, r_{t+1}) \equiv \mathbb{E}_t[(c_{t+1} - \mathbb{E}_t[c_{t+1}])(r_{t+1} - \mathbb{E}_t[r_{t+1}])]. \tag{4.29}$$

First, consider the case when the rate of return is constant, that is,  $\varepsilon_{r,t} = 0 \forall t$ . This implies that  $\mathbb{C}_t(c_{t+1}, r_{t+1}) = 0$ . Also, suppose that the discount rate exceeds the average rate of return, which – by itself – would induce agents to run down their wealth and then take on debt. Uncertainty increases the right-hand side of the Euler equation and dampens the desire of agents to consume more in this period than the next and to reduce wealth levels. Would this mean that the inequality is reversed and that it is optimal to save a lot and increase future consumption levels. The answer is no. The magnitude of the uncertainty effect depends on the value of  $\mathbb{E}_t[c_{t+1}]^{-\gamma-2}$ . Consequently, expected consumption is high enough and  $\mathbb{E}_t[c_{t+1}]^{-\gamma-2}$  low enough, then the uncertainty effect is less important.

If the covariance between unexpected changes in  $c_{t+1}$  and  $r_{t+1}$  is negative—that is, bonds act as a hedge—then the covariance has a similar dampening role. However, if this covariance is positive, then stochastic rates of return would reinforce the desire of the agent to keep on reducing consumption over time.

## 4.7 Appendix B - Accuracy of the projections solution

In this section, we document that the solution generated by the projection algorithm is accurate. It is important to establish this, since the projection solution is used as the benchmark against which the other solutions are compared.

The projection solution uses 8,405,000 grid points; 41 for both  $r$  and  $y$  and 5,000 for  $w_{-1}$ . We use the Dynamic Euler-equation accuracy test, described in Den Haan (2010). The test compares the generated time series with the series that are obtained by explicitly solving the first-order conditions, using the approximation only to calculate next period's wealth choice.<sup>31</sup> This test is more stringent than the standard Euler-equation accuracy test, since it would detect if small errors accumulate in a simulation. Figure 4.6 documents that the two series are very close to each other. Table 4.5 reports some key statistics. Both the figure and the table indicate that the solution is very accurate.

Table 4.5: Accuracy Test

	mean absolute error	maximum absolute error
consumption		
as fraction of $\bar{y}$	$9.2e - 5$	$4.0e - 4$
as fraction of $SD(w_t)$	$1.5e - 5$	$6.7e - 5$
wealth		
as fraction of $\bar{y}$	$2.2e - 3$	$9.4e - 3$
as fraction of $SD(w_t)$	$3.7e - 3$	$1.6e - 3$

Notes: This table reports the errors of the dynamic Euler equation accuracy test using a simulation of 80,000 observations.

<sup>31</sup>In particular, the numerical solution is not used to determine the amount of wealth carried over into the next period.

## 4.8 Appendix C - The CRW procedure

The CRW procedure starts out with the second-order approximation of the Euler equation, which is given by

$$c_t^{-\gamma} = \beta \begin{bmatrix} \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma} \\ +\gamma(\gamma+1)\beta \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma-2} \mathbb{V}_t(c_{t+1}) \\ -\beta\gamma \mathbb{E}_t[c_{t+1}]^{-\gamma-1} \mathbb{C}_t(c_{t+1}, r_{t+1}) \end{bmatrix}, \quad (4.30)$$

where  $\mathbb{V}_t(c_{t+1})$  and  $\mathbb{C}_t(c_{t+1}, r_{t+1})$  are the variance of  $c_{t+1}$  and the covariance of  $c_{t+1}$  and  $r_{t+1}$ , respectively, both conditional on period- $t$  information. In addition, the budget constraint, and the laws of motion for  $y_t$  and  $r_t$  are needed. That is

$$c_t + w_t = y_t + w_{t-1}r_t, \quad (4.31)$$

$$y_t = (1 - \rho_y)\bar{y} + \rho_y y_{t-1} + \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, \sigma_y^2), \quad (4.32a)$$

$$r_t = (1 - \rho_r)\bar{r} + \rho_r r_{t-1} + \varepsilon_{r,t}, \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2). \quad (4.32b)$$

The objective of the CRW procedure is to find an approximation to the savings decision that is linear in levels. The latter is given by

$$w_t = \bar{w} + W_w \hat{w}_{t-1} + W_y \hat{y}_t + W_r \hat{r}_t. \quad (4.33)$$

Using the budget constraint and this linear approximation for  $w_t$ , one can derive the second-order terms in the approximated Euler equation. The results are as follows:

$$\mathbb{V}_t(c_{t+1}) = ((1 - W_y)^2 \sigma_y^2) + (W_r^2 + w_t^2 - 2w_t W_r) \sigma_r^2, \text{ and} \quad (4.34)$$

$$\mathbb{C}_t(c_{t+1}, r_{t+1}) = (w_t - W_r) \sigma_r^2. \quad (4.35)$$

The idea of the CRW algorithm is to find a first-order perturbation solution to the system of equations consisting of equations (4.30), (4.31), (4.32), (4.34), and (4.35), which is consistent with the definitions of  $\mathbb{V}_t(c_{t+1})$  and  $\mathbb{C}_t(c_{t+1}, r_{t+1})$  given in equations (4.34) and (4.35). One could find the values for  $\bar{w}$ ,  $W_r$ ,  $W_y$  using an equation solver or with the following iterative procedure. Start with initial values for  $W_r$  and  $W_y$  and denote these by  $\widetilde{W}_r$  and  $\widetilde{W}_y$ . If we use these values to calculate  $\mathbb{V}_t(c_{t+1})$  and  $\mathbb{C}_t(c_{t+1}, r_{t+1})$  and substitute the expressions for  $\mathbb{V}_t(c_{t+1})$  and  $\mathbb{C}_t(c_{t+1}, r_{t+1})$  into equation (4.30), we get the CRW system of equations:

$$c_t^{-\gamma}$$

$$\begin{aligned}
&= \\
&\left[ \begin{aligned} &\beta \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma} \\ &+ \gamma(\gamma+1) \beta \mathbb{E}_t[r_{t+1}] \mathbb{E}_t[c_{t+1}]^{-\gamma-2} \left( \begin{aligned} &\left( (1 - \widetilde{W}_y)^2 \sigma_y^2 \right) \\ &+ \left( \widetilde{W}_r^2 + w_t^2 - 2w_t \widetilde{W}_r \right) \sigma_r^2 \end{aligned} \right) \\ &- \beta \gamma \mathbb{E}_t[c_{t+1}]^{-\gamma-1} \left( w_t - \widetilde{W}_r \right) \sigma_r^2 \end{aligned} \right], \tag{4.36}
\end{aligned}$$

$$c_t + w_t = y_t + r_t w_{t-1}, \tag{4.37}$$

$$y_t = (1 - \rho_y) \bar{y} + \rho_y y_{t-1} + \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, \sigma_y^2), \tag{4.38}$$

$$r_t = (1 - \rho_r) \bar{r} + \rho_r r_{t-1} + \varepsilon_{r,t}, \quad \varepsilon_{r,t} \sim N(0, \sigma_r^2). \tag{4.39}$$

For given values of  $\widetilde{W}_r$  and  $\widetilde{W}_y$ , this system of equations has a steady state and regular perturbation techniques can be used to calculate  $\widetilde{w}$ ,  $\widetilde{W}_r$ , and  $\widetilde{W}_y$ . The values for  $\widetilde{W}_r$  and  $\widetilde{W}_y$  can then be used to update equation (4.36). This process is continued until convergence.

## 4.9 Appendix D - Details of the ET procedure

In this section, we give detailed information about the implementation of the ET algorithm. The discussion follows the Matlab program closely.<sup>32</sup> To increase transparency, we replace some of the names used in the program with more informative mathematical symbols.<sup>33</sup>

### Define key variables and preliminaries

- The Gauss-Hermite nodes and weights (scaled by  $\sqrt{\pi}$ ) are stored in the  $(N \times 1)$  vectors  $X$  and  $P$ , respectively. We assume that the number of nodes is the same for both random variables.
- The following list of variables are declared to be symbolic variables:  $w, r, w_{-1}, y, \tilde{r}, \tilde{w}, \tilde{y}, \phi_0, \phi_r, \phi_w, \phi_y, \phi_{rr}, \phi_{ww}, \phi_{yy}, \phi_{rw}, \phi_{ry}, \phi_{wy}$ . The last six are only used when implementing second-order ET.
- Define next period's realizations of the two random variables, that is,  $r_{+1}$  and  $y_{+1}$ , as functions of the possible outcomes, i.e., the Gauss-Hermite nodes:

$$r_{+1} = @ (X) \quad r_{+1} = (1 - \rho_r) \bar{r} + \rho_r r + \sqrt{2\sigma_r} X, \quad (4.40)$$

$$y_{+1} = @ (X) \quad y_{+1} = (1 - \rho_y) \tilde{y} + \rho_y y + \sqrt{2\sigma_y} X. \quad (4.41)$$

Note that  $r_{+1}$  and  $y_{+1}$  are not just functions of  $X$ , but also functions of  $r$  and  $y$ .

- Define the left-hand side of the Euler equation, that is,  $c^{-\gamma}$ :

$$L = (y + rw_{-1} - w)^{-\gamma}. \quad (4.42)$$

- The key variable is the Euler equation error, which is equal to

$$\begin{aligned} & Q_0(w, w_{-1}, r, y; \Phi) \\ & = \\ & -1 + \frac{\beta}{L} \sum_{i=1}^N \sum_{j=1}^N r_{+1}(X) \left[ \begin{array}{c} y_{+1}(X(j)) + r_{+1}(X(i)) w \\ -\tilde{h}(r_{+1}, w, y_{+1}; \Phi) \end{array} \right]^{-\gamma} P(i) P(j), \end{aligned} \quad (4.43)$$

<sup>32</sup>The program is available at [sites.google.com/site/michalkobielarz/code](https://sites.google.com/site/michalkobielarz/code).

<sup>33</sup>In particular, `lhs` in the code stands for  $L$ , i.e., the left-hand side of the Euler equation; `wlag` in the code stands for wealth chosen previous period, that is,  $w_{-1}$ ; `wconstm` stands for  $\phi_0$ ; `rm` stands for  $\tilde{r}$ ; `wm` stands for  $\tilde{w}$ ; `ym` stands for  $\tilde{y}$ ; `phi_i` stands for  $\phi_i$  with  $i \in \{\mathbf{r}, \mathbf{w}, \mathbf{y}\}$ ; and `phi_ij` stands for  $\phi_{i,j}$  with  $i, j \in \{\mathbf{r}, \mathbf{w}, \mathbf{y}\}$ ; `eq`

stands for  $Q(\cdot)$ , the Euler-equation error term; `eq_x` stands for  $\partial Q / \partial x$  with  $\mathbf{x} \in \{\mathbf{w}, \mathbf{r}, \mathbf{w}, \mathbf{y}\}$ ; `eq_xz` stands for  $\partial^2 Q / \partial^2 xz$  with  $\mathbf{x}, \mathbf{z} \in \{\mathbf{w}, \mathbf{r}, \mathbf{w}, \mathbf{y}\}$ . In both, the first and second order derivatives, the modifier  $w$  stands for previous period wealth,  $w_{-1}$ , and the modifier  $W$  stands for this period wealth,  $w$ .



where  $\Phi$  contains all the coefficients of the approximating functional form. In the program,  $Q(\cdot)$  is calculated using a double "for loop".

**First-order ET.** When using first-order ET, the function  $\tilde{h}(\cdot)$  is given by

$$\tilde{h}(r_{+1}, w, y_{+1}; \Phi) = \phi_0 + \phi_r(r_{+1}(X) - \tilde{r}) + \phi_w(w - \tilde{w}) + \phi_y(y_{+1}(X) - \tilde{y}) \quad (4.44)$$

where

$$\Phi = [\phi_0, \phi_r, \phi_w, \phi_y, \tilde{r}, \tilde{w}, \tilde{y}]. \quad (4.45)$$

A key aspect of ET is that all elements of  $\Phi$  depend on the current values of  $r$ ,  $w_{-1}$ , and  $y$ .

The model says that the error term defined in equation (4.43) should be equal to zero for all  $r$ ,  $w_{-1}$ , and  $y$ , that is,

$$Q_0(w, w_{-1}, r, y; \Phi) = 0. \quad (4.46)$$

Differentiating this equation with respect to  $r$ ,  $w_{-1}$ , and  $y$  gives

$$\frac{\partial Q_0}{\partial w} \frac{\partial w}{\partial r} + \frac{\partial Q_0}{\partial r} = 0, \quad (4.47a)$$

$$\frac{\partial Q_0}{\partial w} \frac{\partial w}{\partial w_{-1}} + \frac{\partial Q_0}{\partial w_{-1}} = 0, \quad (4.47b)$$

$$\frac{\partial Q_0}{\partial w} \frac{\partial w}{\partial y} + \frac{\partial Q_0}{\partial y} = 0, \quad (4.47c)$$

With a symbolic toolbox, one defines  $Q_0$  as a function, exactly as it is defined in equation (4.43). Next, one obtains the partial derivatives as functions of  $w$ ,  $w_{-1}$ ,  $r$ , and  $y$  by using the symbolic toolbox' differentiation command.<sup>34</sup> *After* the derivatives have been calculated, one would impose that

$$\phi_0 = w, \quad (4.48)$$

$$\tilde{r} = r, \quad (4.49)$$

$$\tilde{w} = w_{-1}, \quad (4.50)$$

$$\tilde{y} = y. \quad (4.51)$$

That is, the Taylor series expansion to characterize next period's choice is around today's state variables and the function value at those outcomes is, of course, equal to this period's choice. The derivatives of this Taylor series expansion are equal to the derivatives of this

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<sup>34</sup>In Matlab, `Q_r=diff(Q_0,r)` generates the derivative of `Q_0` with respect to `r`.

period's choice. That is,

$$\phi_r = \frac{\partial w}{\partial r}, \quad (4.52a)$$

$$\phi_w = \frac{\partial w}{\partial w_{-1}}. \quad (4.52b)$$

$$\phi_y = \frac{\partial w}{\partial y}. \quad (4.52c)$$

After these substitutions, we get the following system of four equations in the four unknowns  $w$ ,  $\phi_r$ ,  $\phi_w$ , and  $\phi_y$  – for given values of  $r$ ,  $w_{-1}$ , and  $y$ :<sup>35</sup>

$$0 = Q_0(w, w_{-1}, r, y; \tilde{\Phi}), \quad (4.53a)$$

$$0 = Q_r(w, w_{-1}, r, y; \tilde{\Phi}), \quad (4.53b)$$

$$0 = Q_w(w, w_{-1}, r, y; \tilde{\Phi}), \quad (4.53c)$$

$$0 = Q_y(w, w_{-1}, r, y; \tilde{\Phi}), \quad (4.53d)$$

where

$$\tilde{\Phi} = [\phi_r, \phi_w, \phi_y]. \quad (4.54)$$

**Second-order ET.** When using second-order ET, the function  $\tilde{h}(\cdot)$  is given by

$$\begin{aligned} & \tilde{h}(r_{+1}, w, y_{+1}; \Phi) \\ &= \\ & \phi_0 + \phi_r(r_{+1}(X) - \tilde{r}) + \phi_w(w - \tilde{w}) + \phi_y(y_{+1}(X) - \tilde{y}) \\ & + \frac{1}{2}\phi_{rr}(r_{+1}(X) - \tilde{r})^2 + \frac{1}{2}\phi_{ww}(w - \tilde{w})^2 + \frac{1}{2}\phi_{yy}(y_{+1}(X) - \tilde{y})^2 \\ & + \phi_{rw}(r_{+1}(X) - \tilde{r})(w - \tilde{w}) + \phi_{ry}(r_{+1}(X) - \tilde{r})(y_{+1}(X) - \tilde{y}) \\ & + \phi_{wy}(w - \tilde{w})(y_{+1}(X) - \tilde{y}) \end{aligned} \quad (4.55)$$

where

$$\Phi = [\phi_0, \phi_r, \phi_w, \phi_y, \phi_{rr}, \phi_{rw}, \phi_{ry}, \phi_{wy}, \tilde{r}, \tilde{w}, \tilde{y}]. \quad (4.56)$$

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<sup>35</sup>For this system of equations, the nonlinear-equation solver is keen to set  $\phi_w$  to values close to  $r$  and  $\phi_y$  to values close to 1. Since  $r > 1$ , this implies an explosive solution. It is easy to show that  $\phi_w = r$  and  $\phi_y = 1$  are exact solutions to equations (4.53c) and (4.53d) when  $\sigma_r = 0$ . To avoid the nonlinear-equation solver choosing or moving towards this possibility, we divide equation (4.53c) by  $r - \phi_w$  and equation (4.53d) by  $1 - \phi_y$ .

The additional six equations and associated error terms are given by

$$0 = Q_{ij}(w, w_{-1}, r, y; \Phi) \quad i, j \in \{r, w, y\} \quad (4.57)$$

$$Q_{rr}(w, w_{-1}, r, y; \Phi) = \frac{\partial Q}{\partial w} \phi_{rr} + 2 \frac{\partial^2 Q}{\partial w \partial r} \phi_r + \frac{\partial^2 Q}{\partial w^2} \phi_r^2 + \frac{\partial^2 Q}{\partial r^2}, \quad (4.58)$$

$$Q_{ww}(w, w_{-1}, r, y; \Phi) = \frac{\partial Q}{\partial w} \phi_{ww} + 2 \frac{\partial^2 Q}{\partial w \partial w_{-1}} \phi_w + \frac{\partial^2 Q}{\partial w^2} \phi_w^2 + \frac{\partial^2 Q}{\partial w_{-1}^2}, \quad (4.59)$$

$$Q_{yy}(w, w_{-1}, r, y; \Phi) = \frac{\partial Q}{\partial w} \phi_{yy} + 2 \frac{\partial^2 Q}{\partial w \partial y} \phi_y + \frac{\partial^2 Q}{\partial w^2} \phi_y^2 + \frac{\partial^2 Q}{\partial y^2}, \quad (4.60)$$

$$Q_{rw}(w, w_{-1}, r, y; \Phi) = \left[ \begin{array}{c} \frac{\partial Q}{\partial w} \phi_{rw} + \frac{\partial^2 Q}{\partial w \partial w_{-1}} \phi_r + \frac{\partial^2 Q}{\partial w \partial r} \phi_w \\ + \frac{\partial^2 Q}{\partial w^2} \phi_r \phi_w + \frac{\partial^2 Q}{\partial r \partial w_{-1}} \end{array} \right], \quad (4.61)$$

$$Q_{ry}(w, w_{-1}, r, y; \Phi) = \left[ \begin{array}{c} \frac{\partial Q}{\partial w} \phi_{ry} + \frac{\partial^2 Q}{\partial w \partial y} \phi_r + \frac{\partial^2 Q}{\partial w \partial r} \phi_y \\ + \frac{\partial^2 Q}{\partial w^2} \phi_r \phi_y + \frac{\partial^2 Q}{\partial r \partial y} \end{array} \right], \quad (4.62)$$

$$Q_{wy}(w, w_{-1}, r, y; \Phi) = \left[ \begin{array}{c} \frac{\partial Q}{\partial w} \phi_{wy} + \frac{\partial^2 Q}{\partial w \partial y} \phi_w + \frac{\partial^2 Q}{\partial w \partial w_{-1}} \phi_y \\ + \frac{\partial^2 Q}{\partial w^2} \phi_w \phi_y + \frac{\partial^2 Q}{\partial w_{-1} \partial y} \end{array} \right]. \quad (4.63)$$

These equations are obtained by differentiating equation (4.43) twice and imposing that

$$\begin{aligned} \phi_{rr} &= \frac{\partial^2 w}{\partial r^2}, \phi_{ww} = \frac{\partial^2 w}{\partial w_{-1}^2}, \phi_{yy} = \frac{\partial^2 w}{\partial y^2}, \\ \phi_{rw} &= \frac{\partial^2 w}{\partial r \partial w_{-1}}, \phi_{ry} = \frac{\partial^2 w}{\partial r \partial y}, \phi_{wy} = \frac{\partial^2 w}{\partial w_{-1} \partial y}. \end{aligned} \quad (4.64)$$

## 4.10 Appendix E - Results for other parameter values

The analysis in the main text is based on values for the standard deviations of the innovations that are lower than the ones used in Coeurdacier et al. (2011). The reason for our choice of parameter values is that the model generates very complex nonlinear "escape" dynamics at the original CRW parameter values. More precisely, we find that the solution for wealth generated by an accurate projections method hits the imposed upper bound on wealth when the sample is long enough even when that upper bound is extremely high.<sup>36</sup> This is documented in figure 4.7, which plots parts of the projection and CRW time paths at the original CRW parameter values. The figure has three panels, each corresponding with a different upper bound on wealth imposed by the projection algorithm. The three values considered are 35, 100, and 1000. The figure documents that increasing the upper bound is not effective in making it not binding. In fact, if the upper bound is increased from 100 to 1000 it actually becomes binding more often in this part of the state space.

Escape dynamics occur at high values of the rate of return, but it is not due to a few extreme values for the rate of return, since the same pattern of results is found when the realizations of  $r_t$  are constrained not to be further away from their mean than two standard deviations. As documented in figure 4.7, the CRW solution does not capture these escape dynamics. The ET algorithm also has difficulty obtaining solutions at many points in the state space.

This type of behavior indicates that the model may not have a stationary solution at those parameter values unless one does impose an upper bound.<sup>37</sup> Projection methods can incorporate such a bound, but this is not possible for CRW and ET. Therefore, we prefer not to do a formal comparison of different solution methods at parameter values where we observe this type of escape dynamics.

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<sup>36</sup>At higher values for  $\sigma_r$  and  $\sigma_y$ , it is important for the stability of the projection algorithm that an upper bound is imposed.

<sup>37</sup>It is possible that there is a stationary solution despite the escape dynamics, but that the upper bounds considered here are still not high enough. Imposing the upper bound flattens the policy function for wealth at high interest rates even at values of wealth which are not that close to the upper bound, which makes it even more surprising that the upper bound is reached during simulations.

## 4.11 Appendix F - The neoclassical growth model

In this section, we make clear how easy it is to program the ET algorithm by providing the Matlab code that uses first-order ET to solve the neoclassical growth model, a model that most readers will be familiar with. The program uses the Matlab symbolic toolbox to calculate derivatives. The user also needs a subroutine to generate the Gauss-Hermite quadrature nodes and weights. These are readily available and we provide one at [sites.google.com/site/michalkobielarz/code](https://sites.google.com/site/michalkobielarz/code). This website also contains the code displayed here and the program to solve the same model with second-order ET.

```
clear;
% This program solves the stochastic Ramsey growth model using the
% ET-algorithm by Den Haan, Kobielarz, and Rendahl (2015).
% Declare parameters for the model

alpha = 1/3;          % Capital share of income: 1/3.
beta = 1.04^(-1/4);   % Discount factor: 4% annual interest rate.
delta = 0.025;        % Depreciation rate.
gamma = 10;           % Coefficient of relative risk aversion.
sigma = 0.03;         % Standard deviation of productivity shock.
rho = 0.95;           % Persistence of technology shock.

% Implied steady state

kss = ((1/beta+delta-1)/alpha)^(1/(alpha-1));
css = kss^(alpha)-delta*kss;

% Information regarding the shock

N_quad = 5;          % Number of nodes for the quadrature (5 goes a long way).
[Z W] = hernodes(N_quad); % Generate nodes and weights.
W = pi^(-1/2)*W;      % Normalize weights.
Z = Z*sqrt(2)*sigma;   % Normalize nodes.

% Declar symbolic variables

syms k c z phi_k phi_z kt ct zt

kp = exp(z)*k^(alpha)+(1-delta)*k-c;
zp = rho*z+Z;

% Define the Euler Equation

EE = c-(W'*( beta*(1+exp(zp)*alpha*kp^(alpha-1)-delta) ...
    .*(ct+phi_k*(kp-kt)+phi_z*(zp-zt)).^(-gamma) ))^(-1/gamma);
```

```

% Take derivatives

dk = diff(EE,k);
dz = diff(EE,z);
dc = diff(EE,c);

% The implicit function theorem reveals that
% the following equations should hold:

Ek = phi_k+dk./dc;
Ez = phi_z+dz./dc;

% The collection of equations to be solved is therefore

E = [EE;Ek;Ez];
E = subs(E,[kt zt ct],[k z c]);

% Convert into a matlab function

E = matlabFunction(E,'vars',{[c;phi_k;phi_z],k,z});

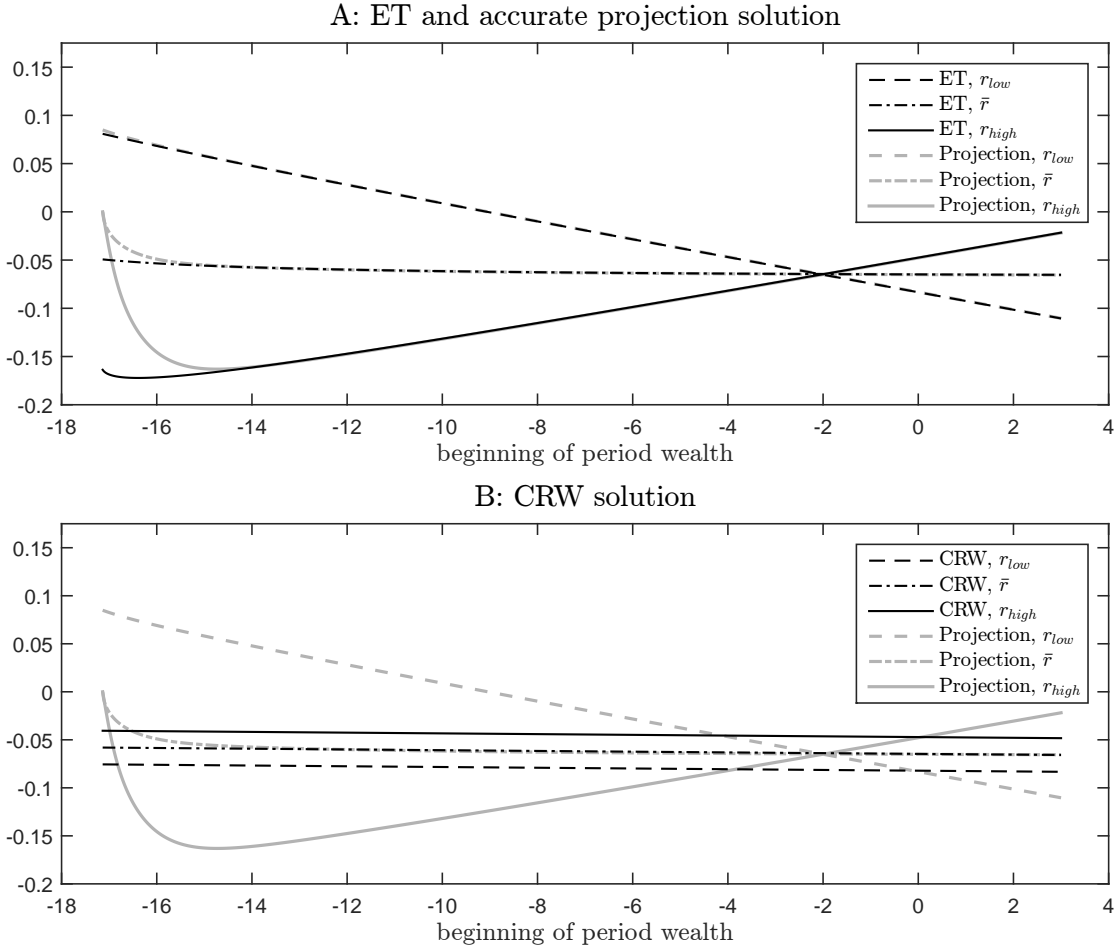
% Solve the problem along a stochastic simulation.

T = 500;
k0 = zeros(1,T+1);
z0 = zeros(1,T+1);
k0(1) = kss;
z0(1) = 0;
% Initial guess
X = [css;0.9;0.9];
options = optimset('Display','off','TolFun',1e-14,'TolX',1e-14);
rng(20150215,'twister');
e = sigma*randn(T,1);

for t = 1:T
    disp(t)
    E1 = @(x) E(x,k0(t),z0(t));
    X(:,t+1) = fsolve(E1,X(:,t),options);
    k0(t+1) = exp(z0(t))*k0(t)^(alpha)+(1-delta)*k0(t)-X(1,t+1);
    z0(t+1) = rho*z0(t)+e(t);
end
c0 = X(1,:);
% Done.

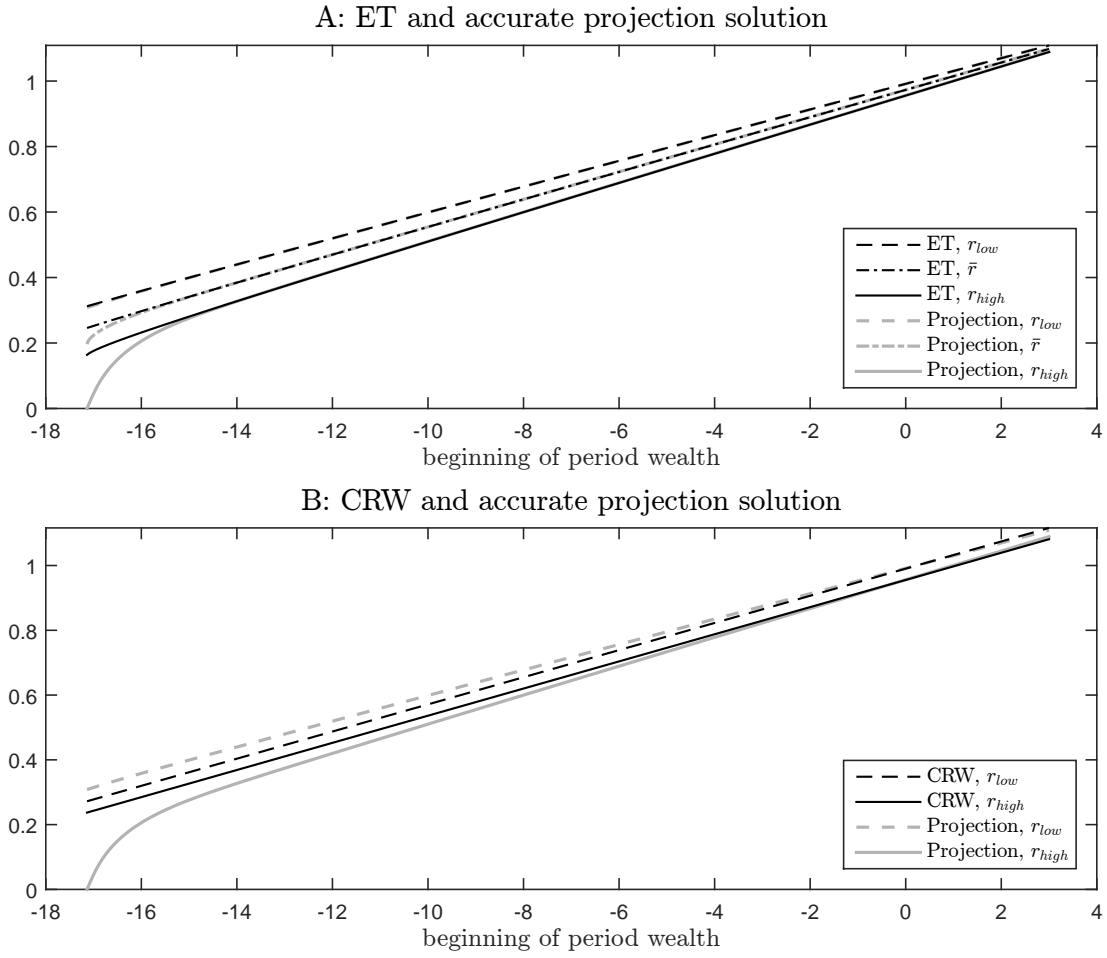
```

Figure 4.1: Wealth Policy Function (low  $y$  value)



Notes: These graphs plots the values of the change in wealth,  $w - w_{-1}$ , according to the indicated algorithm as a function of beginning-of-period wealth,  $w_{-1}$ , when income,  $y$ , is equal to 0.9082. The three values of the realized gross rate of return,  $r$ , are equal to 1.0301, 1.0358, and 1.0415.

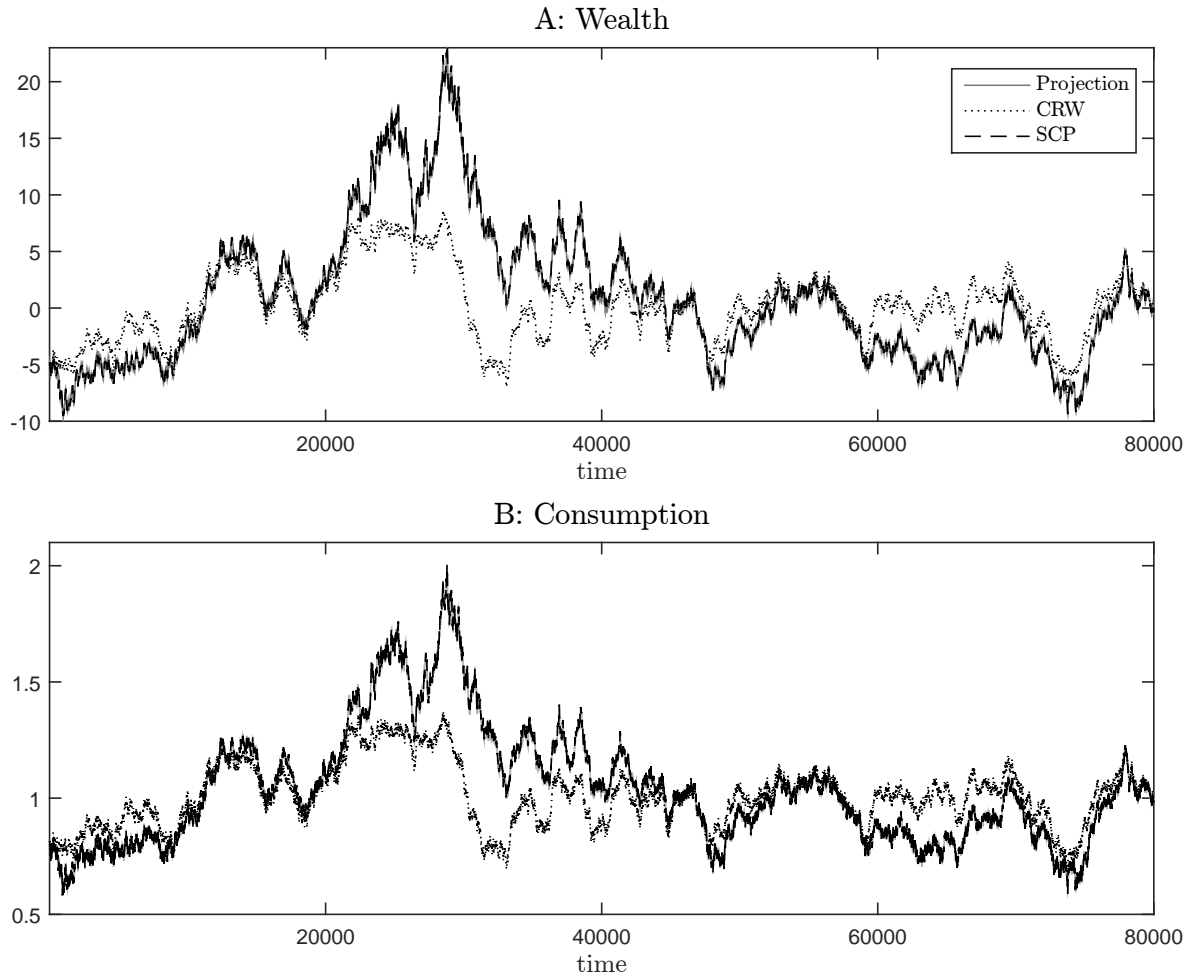
Figure 4.2: Consumption Policy Function (low  $y$  value)



Notes: These graphs plots the values of consumption,  $c$ , according to the indicated algorithm as a function of beginning-of-period wealth,  $w_{-1}$ , when income,  $y$ , is equal to 0.9082. The three values of the realized gross rate of return,  $r$ , are equal to 1.0301, 10358, and 1.0415.

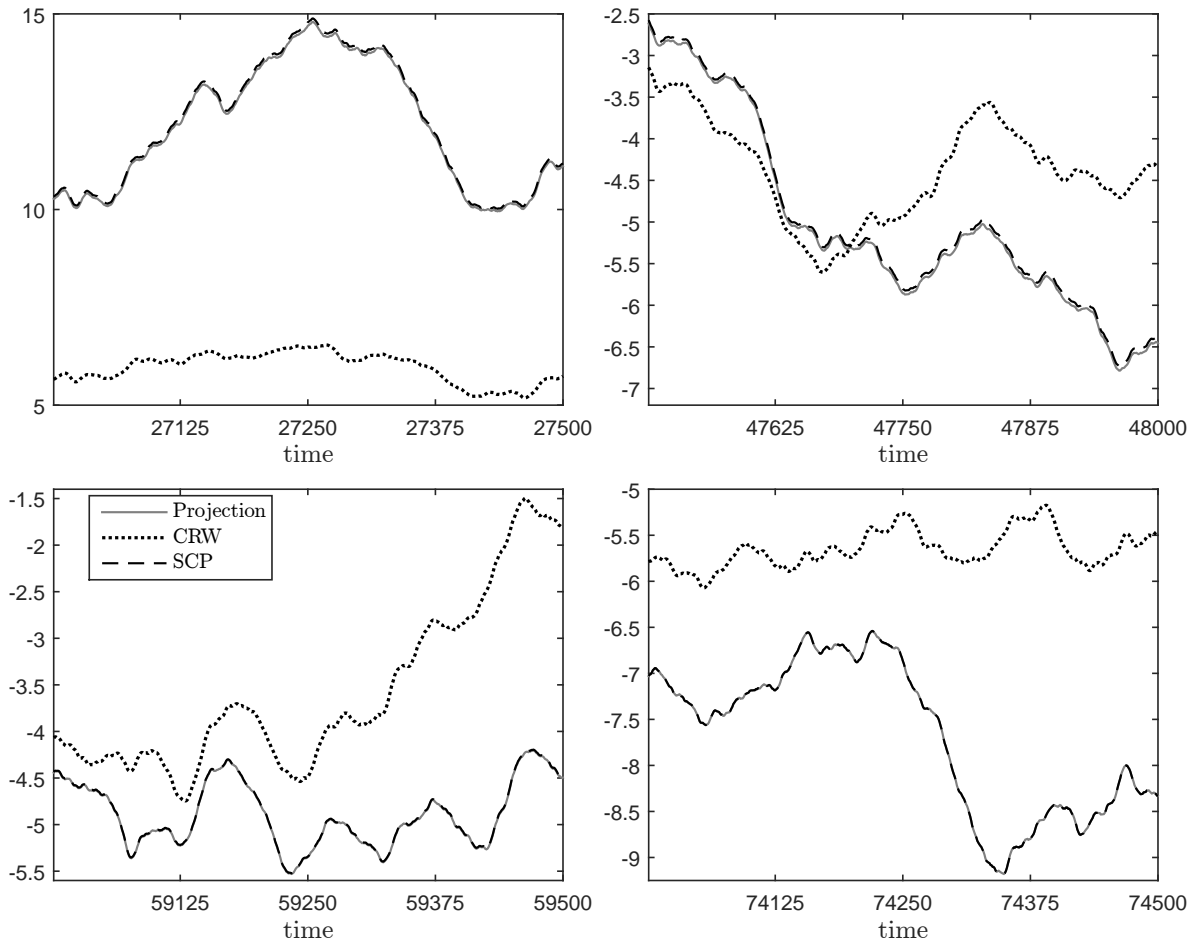


Figure 4.3: Simulated Time Paths (whole sample)



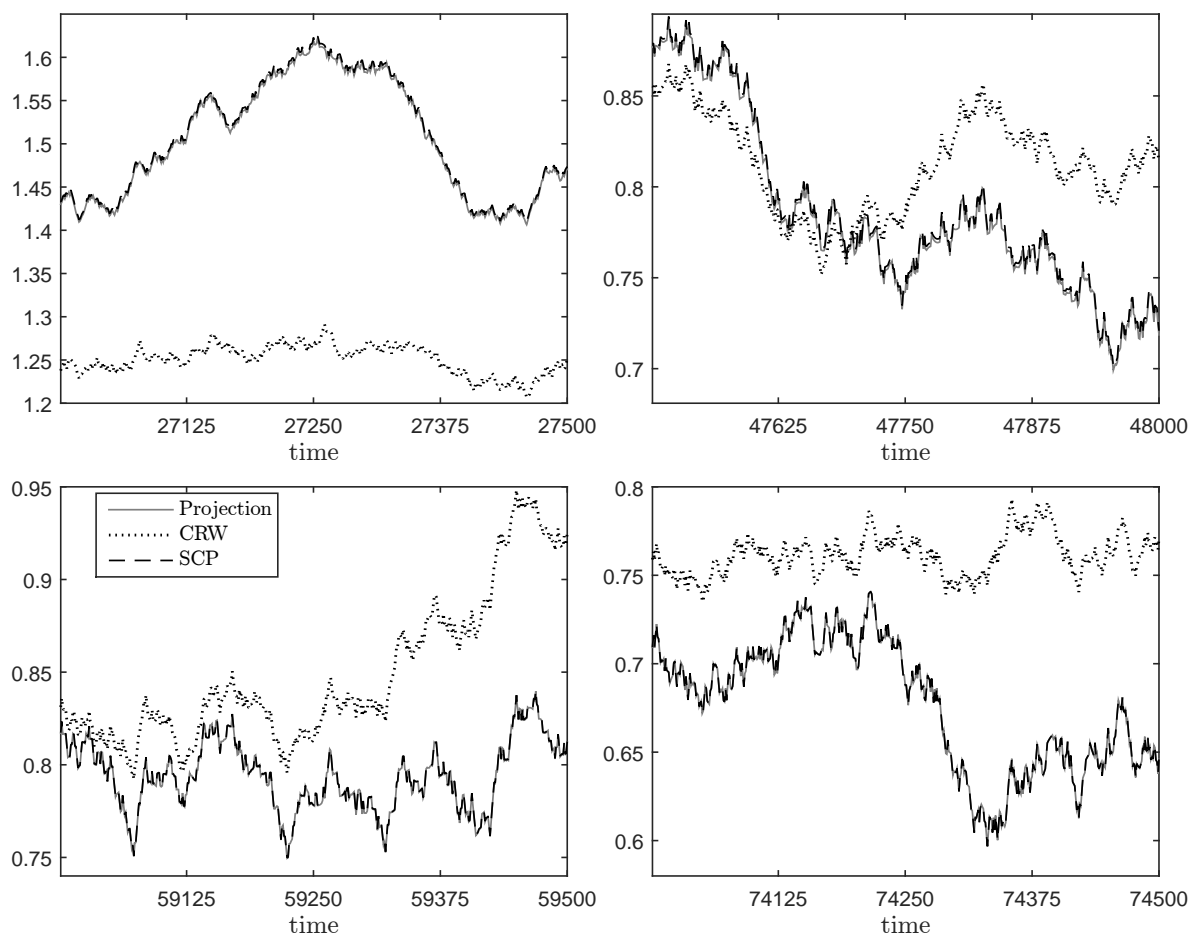
Notes: This graph plots simulated time series generated with the three algorithms using the exact same initial conditions and same draws for the exogenous random variables. Whereas the projection and the ET solution are difficult to distinguish, the CRW solution is clearly different.

Figure 4.4: Selected Short Samples: Wealth



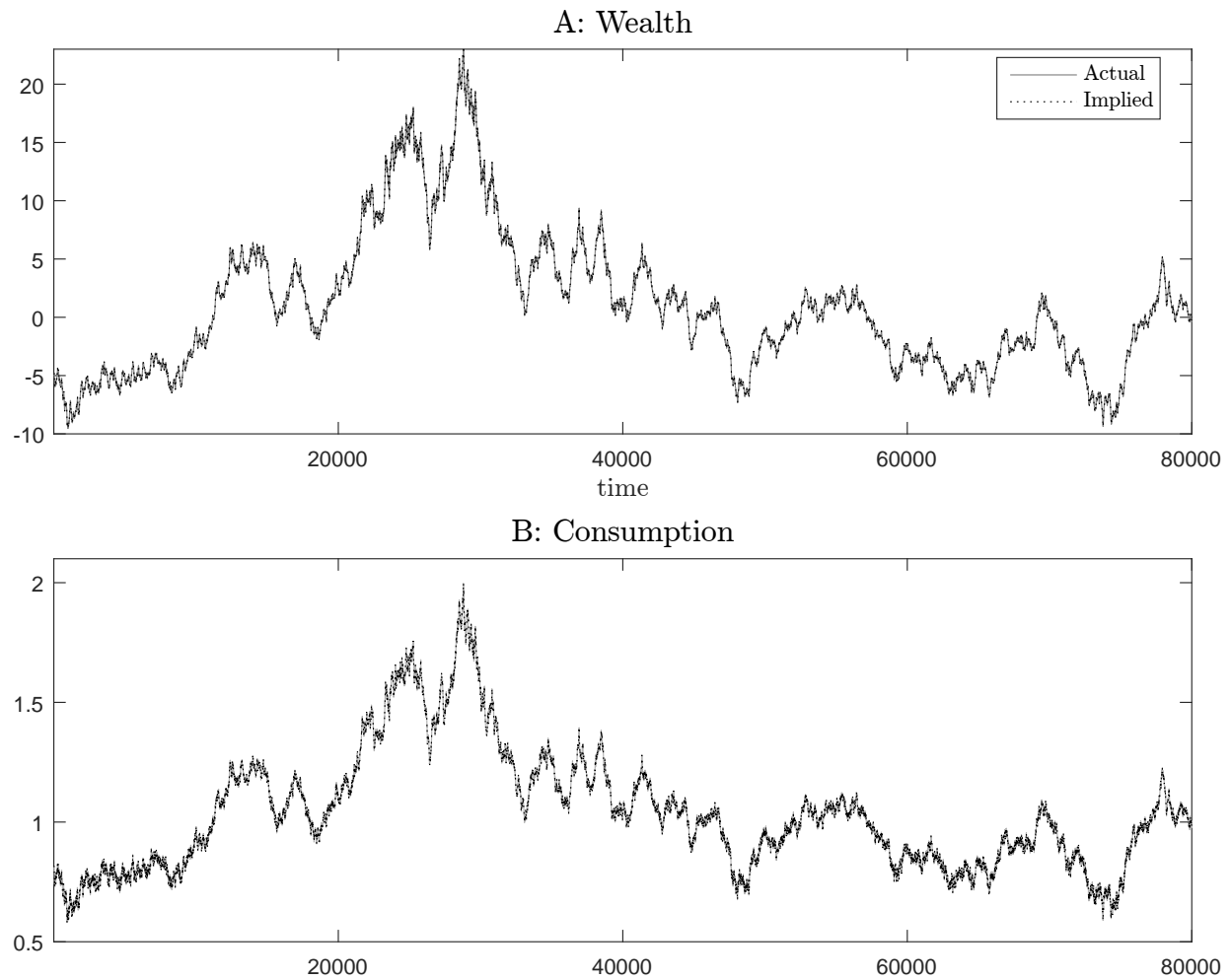
Notes: This graph plots parts of the simulated time series for wealth generated with the three algorithms using the exact same initial conditions and same draws for the exogenous random variables. Whereas the projection and the ET solution are difficult to distinguish, the CRW solution is clearly different.

Figure 4.5: Selected Short Samples: Consumption



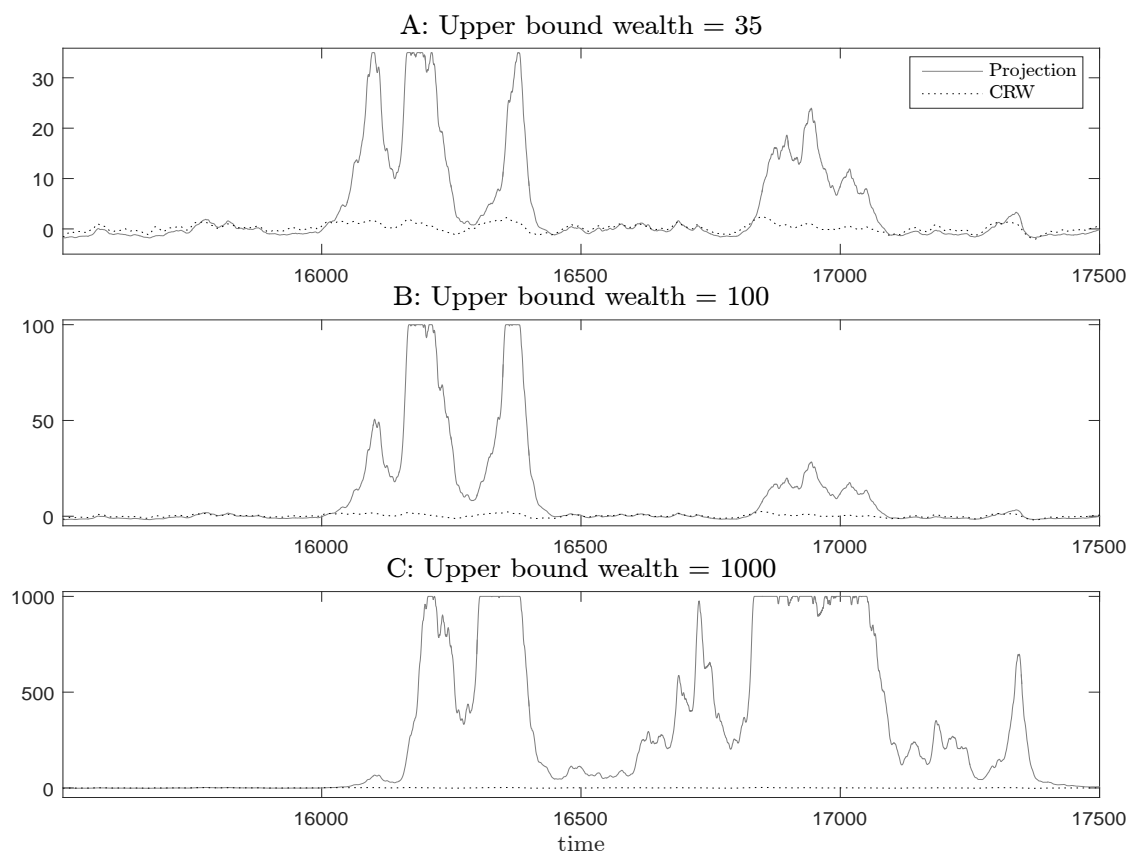
Notes: This graph plots parts of the simulated time series for consumption generated with the three algorithms using the exact same initial conditions and same draws for the exogenous random variables. Whereas the projection and the ET solution are difficult to distinguish, the CRW solution is clearly different.

Figure 4.6: Dynamic Euler-Equation Accuracy Test



Notes: These graphs plot the actual time series for wealth and consumption and the time series that are the explicit solutions to the first-order conditions using the approximation to calculate next period's choice for wealth. The fact that the two series are difficult to distinguish indicates that the solution is accurate.

Figure 4.7: Simulated Wealth Time Series - Original CRW Parameter Values



Notes: This graph plots simulated time series for wealth generated with the projection and the CRW algorithm using the original CRW parameter values. The projection algorithm imposes the indicated upper bound for wealth.

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